SINGLE SURFACE HARDENING MODEL

A System Law to Describe the Foundation Soil Interaction

Aloys Kisse & Kerstin Lesny
University of Duisburg-Essen, Germany http://www.uni-essen.de/grundbau

A thorough understanding of the structure-soil interaction is the basis for a safe and economic design. Hence, it is necessary to ensure that the foundation of a structure, e.g. a shallow foundation, does not fail ensuring a prescribed safety and the displacements are allowable. Because different inconsistent limit states and serviceability states have to be distinguished, the interaction is usually unprecticalbe (Fig. 1). With a system law, which describes the relationship between loading up to failure and corresponding displacements and rotations of the foundation, the distinction between different limit states is no longer be necessary.

The concept of the Single Surface Hardening Model consists of two componentes, a failure condition, which describes the ultimate bearing capacity of the foundation, and a displacement rule, which describes the load displacement behaviour from beginning of loading up to failure of the system.

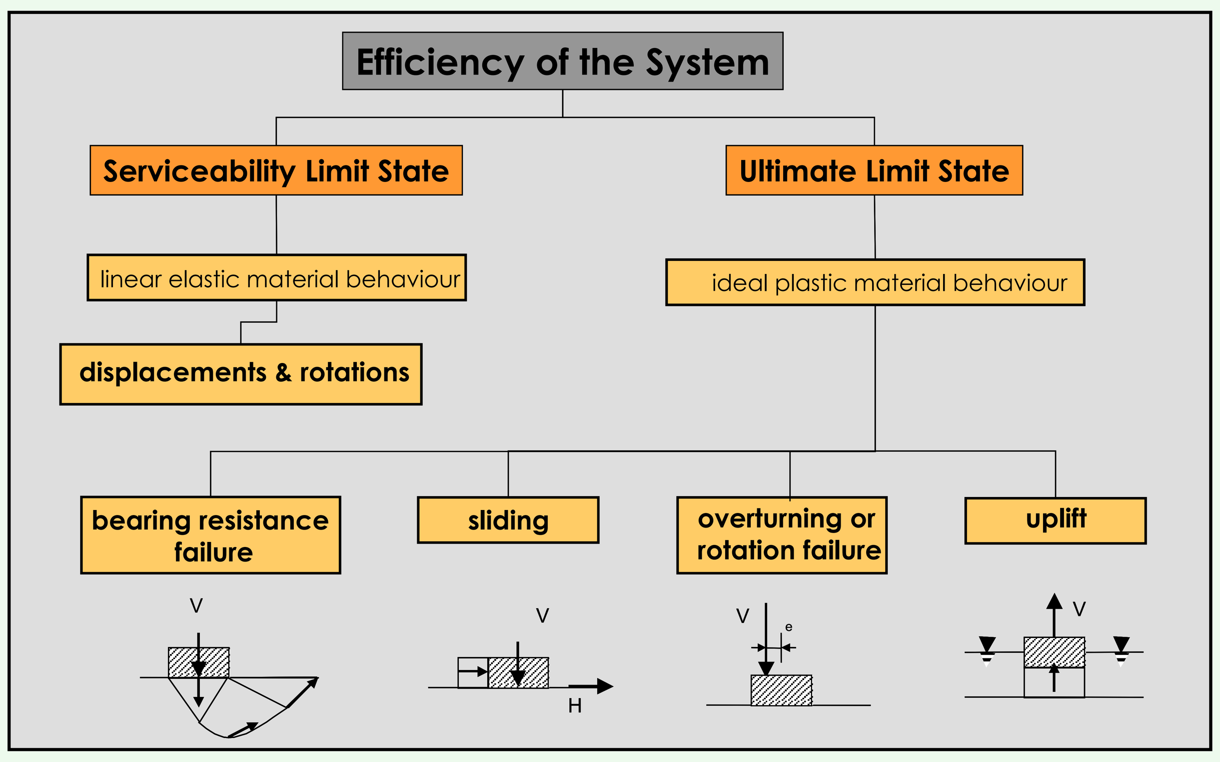

Fig. 1: Today's limit state design

Failure condition

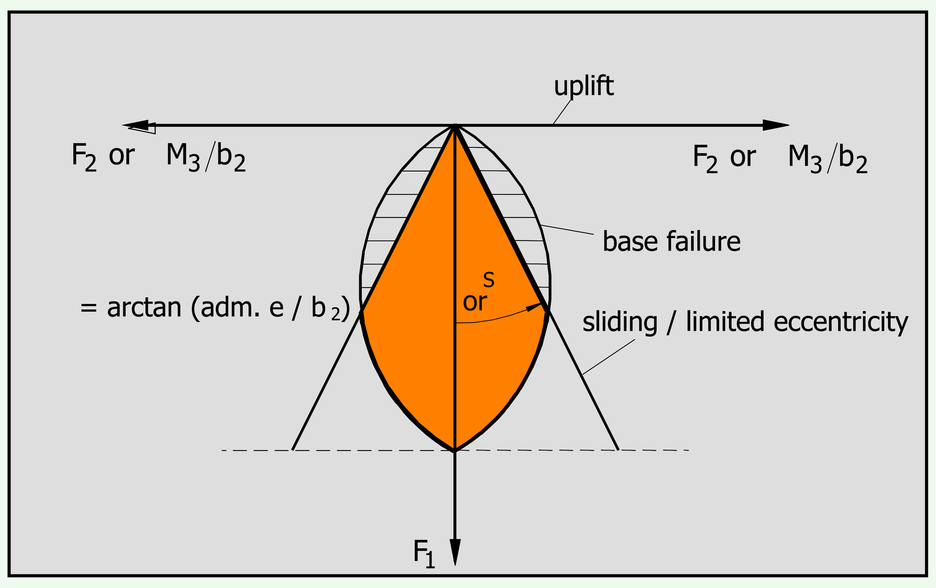

Fig. 2: Interaction Diagram:
Limit state equation in loading space

In analogy to the concept of constitutive laws of plasticity the failure condition consistently describes the ultimate bearing capacity of the foundation similar to a yield condtion. Here the failure condition is formulated for the case of six degrees of freedom (Lesny, 2001):

$$F \quad \frac{F_2^2 \quad F_3^2}{a_1 \; F_{10}^2} \quad \frac{M_1^2}{a_2 \; F_{10}^2} \quad \frac{M_2^2}{a_3 \; b_3 \; F_{10}^2} \quad \frac{M_3^2}{a_3 \; b_2 \; F_{10}^2} \quad \frac{F_1}{F_{10}} \; 1 \; \frac{F_1}{F_{10}} \quad ^2 \quad 0$$

The parameters $a_{1,2,3}$ govern the inclination of the failure surface for small vertical loading where the limit state equations of sliding and overturning previously have been relevant (Fig. 2). The parameter controls the position of the maximum of the failure surface. The quantity F_{10} represents the resistance of a footing under pure vertical loading which can be calculated using traditional bearing resistance formula.

Within this model the former isolated failure modes are integrated, so that the failure condition describes the outer boundary of the resultant and admissible loading which itself is named as failure surface.

Fig. 3, for example, shows the influence of the side ratio $\bar{b}=b_2/b_3$ on the bearing capacity of the foundation for different load combinations.

For these study numerous small scale model tests conducted at our institute over the past years have been analysed. Additionally, model tests described in the literature have been considered as far as suitable.

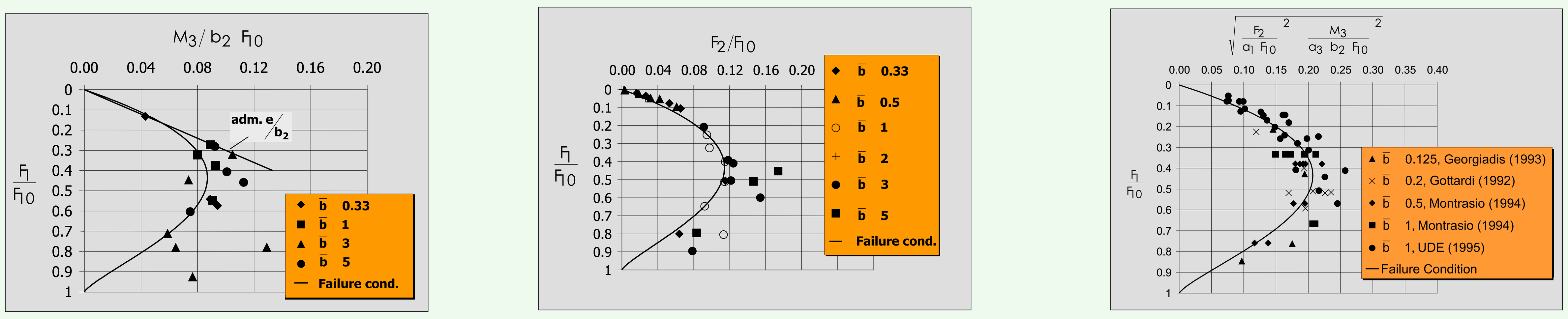

Fig. 3: Failure condition for the basic case (no embedment, no cohesion) :
a) vertical eccentric loading b) inclined centric loading c) combined loading

Displacement rule

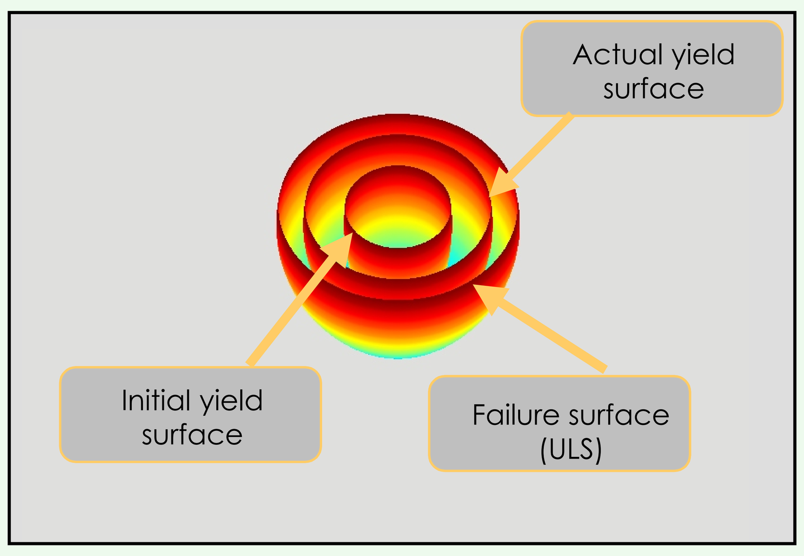

Fig. 4: Isotropic strain hardening concept

The displacements u and rotations of the foundation are caused by loading inside the failure surface. Due to the complex interaction of load components, displacements and rotations the displacement rule has been formulated using the strain hardening plasicity theory with isotropic hardening:

$$d\vec{u} \quad \frac{1}{H} \quad \frac{F}{\vec{Q}}^{T} \quad \frac{G}{\vec{Q}} \quad \vec{Q}$$

Flow condition

$$F \quad \frac{F_2^2 \quad F_3^2}{a_1 \; F_a^2} \quad \frac{M_1^2}{a_2 \; F_a^2} \quad \frac{M_2^2}{a_3 \; b_3 \; F_a^2} \quad \frac{M_3^2}{a_3 \; b_2 \; F_a^2} \quad \frac{F_1}{F_{10}} \; 1 \; \frac{F_1}{F_a} \quad ^2 \quad 0$$

Plastic potential

$$G \quad \frac{F_2^2 \quad F_3^2}{c_1 \; F_b^2} \quad \frac{M_1^2}{c_2 \; F_b^2} \quad \frac{M_2^2}{c_3 \; b_3 \; F_b^2} \quad \frac{M_3^2}{c_3 \; b_2 \; F_b^2} \quad \frac{F_1}{F_{10}} \; 1 \; \frac{F_1}{F_b} \quad ^{\beta \; 2} \quad 0$$

Hardening law

$$F_a \quad \frac{F_1}{F_{10}} \; 1 \; \exp \; \frac{A}{F_{10}^2} \sqrt{u_1^2 \; F_{10}^2}$$

$$A \quad e_{min} \quad _{max} \quad _{min} \quad A_F \; D^3 \; ^m$$

The parameter A describes the initial stiffness of the load displacement relationship. In general the factor A is determined with small scale model tests under centric vertical loading. Since this is not convenient for practical applications, a procedure was developed to determine A from soil mechanical standard tests.

Herein $_{max}$ and $_{min}$ represent the bulk density at minimum and maximum density. The void ratio e_{max} refers to the minimum density of the soil. The relative density D relates the actual density of the soil to the minimum and maximum denisties. The exponent m is derived from an oedometer test with an initial density representing the conditions in the field.

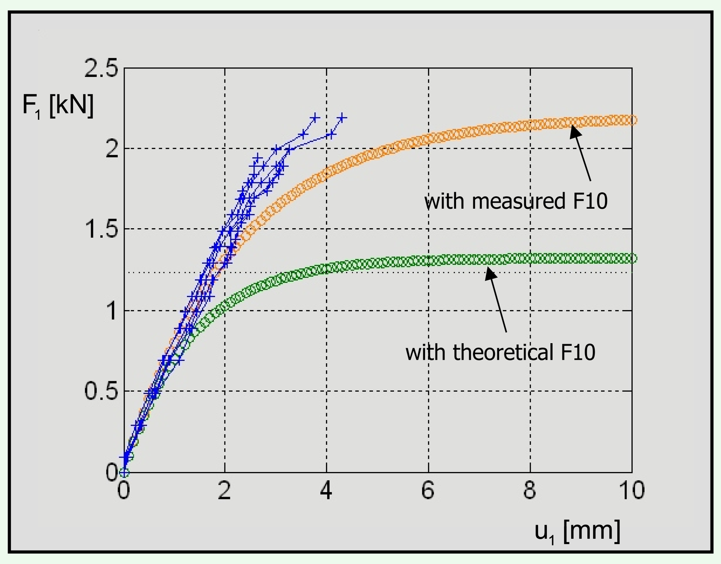

Fig. 5: Comparison of experimental results and theoretical prediction of a small scale model test under centric vertical loading

Fig. 5 shows the experimental result for some small scale model tests under centric vertical loading and a simulation of the same tests with the system law. The failure load F_{10} has been determined here according to the traditional bearing resistance formula and to the failure loads measured in the tests as well. These failure loads are obviously larger than the theoretical values resulting in a steeper load-displacement curve in comparison to the theoretical results.

The simulation of small scale model tests with an eccentric vertical loading using the SSH-Model is shown in Fig. 6. As well as the load case before a significant influence of the way how the failure load F_{10} is determined can be observed.

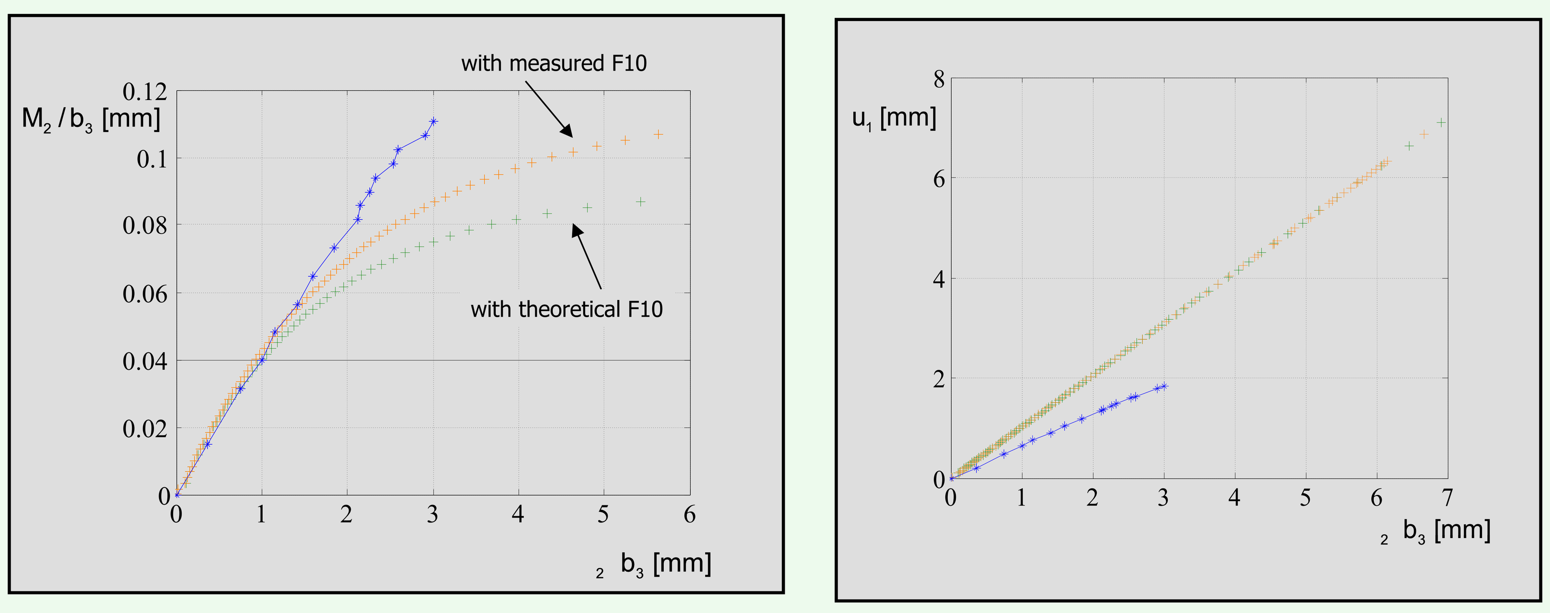

Fig. 6: Simulation of a small scale model test with eccentric loading on dense Essen sand