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Safety of Shallow Foundations in Limit State Design

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Abstract

Important aspects of the limit state design and the partial safety factor method applied to shallow foundations are presented. Some major shortcomings of this concept are pointed out, using the example of a vertical breakwater. A new method is provided which describes consistently the overall relationship between loading and corresponding displacements and rotations of the foundation.

Introduction

The new generation of geotechnical design codes, e. g. pREN 1997-1 (2004) (EC 7), prescribe the limit state design (LSD). Within this design concept several ultimate limit states (ULS) and serviceability limit states (SLS) are investigated. The safety of the structure is calculated with the help of the partial safety factor method. The application of the LSD to shallow foundations includes the separate analysis of different failure modes, e.g. bearing resistance failure or sliding, which describe the complex behaviour of the foundation. This procedure has apparent disadvantages, particularly in the design of foundations under complex loading such as coastal structures. For these structures the actual level of safety can only be approximated if different partial safety factors are applied to different limit states. In the following some of these shortcomings are pointed out by using the example of a vertical breakwater. The results show that the behaviour of a foundation should be defined consistently to predict its safety level reliably. Such a model, which integrates the isolated limit states, is presented at the end of this paper.

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Evaluation of the actual safety of shallow foundations

For foundations under complex loading different failure modes have to be examined for different load combinations within the LSD procedure. For example, the design of vertical caisson breakwaters on a feasibility level includes the investigation of loading in case of still-water level (SWL), wave crest and wave trough (Fig. 1). The failure modes uplift, rotation failure, sliding and bearing resistance failure in the rubble mound or in the subsoil have to be checked. Rotation failure, however, is usually substituted by limiting the eccentricity of the resultant vertical loading to $1/3$ of the foundation width.

Within the EU-MAST III PROVERBS project (Probabilistic design tools for vertical breakwaters, Oumeraci et al., 2001) an extensive parameter study was conducted to analyse the complex interaction between the different load cases and failure modes and the corresponding input parameters. Some results of this study are presented in the following. Details are described in Lesny et al. (2000).

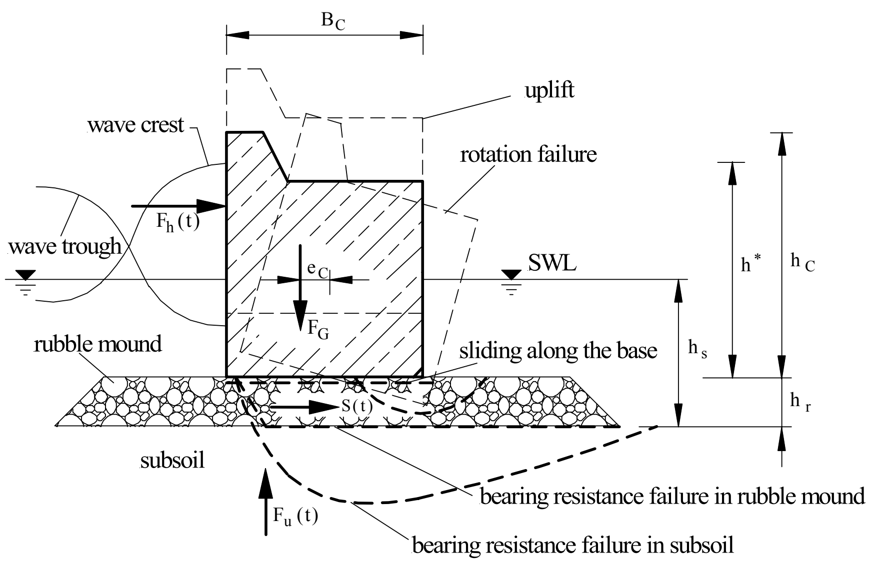

Figure 1. Vertical Breakwater, wave loading and failure modes

The study was confined to the ULS of a vertical breakwater with a thin rubble mound on sandy subsoil (Fig. 1). All input parameters like water and wave parameters, geometric parameters, unit weights and strength parameters of the soil were varied within ranges that cover typical design situations. The variation ranges of the most important design parameters are summarized in Table 1.

Table 1. Important input parameters and their variation range

Parameter	Variation range
significant inshore wave height H_{si}/h_s	$0.0 < H_{si}/h_s \le 0.6$
width of caisson B_C/h_s	design parameter
height of caisson h_C/h_s	$0.8 \cdot h^* \le h_C/h_s \le 1.2 \cdot h^*$
weight of caisson $\alpha_C \cdot \gamma_C/\gamma_w$	$1.4 \le \alpha_C \cdot \gamma_C / \gamma_w \le 2.3$
eccentricity of caisson dead load e_C/B_C	$-0.2 \le e_C / B_C \le 0.2$
friction angle of rubble mound φ'_r	$30° \le \varphi'_r \le 45°$
friction angle of subsoil φ'_s	$25° \le \varphi'_s \le 45°$
REMARKS	
h_s: height of still water level, γ_w : weight of water, $\alpha_C = \text{cross-section}/(B_C \cdot h_C)$	

In the design process the width of the caisson B_C was determined so that none of the limit states of Fig. 1 was reached. The caisson dimensions were limited to $0.5 \le B_{C,crit}/h_C \le 2.0$. Table 2 shows the number of cases in which a certain combination of load case and failure mode determine the required caisson width. Here, only load cases and failure modes which were generally relevant are specified. Other design situations were not critical at all (e. g. loading at SWL, failure due to uplift) or restricted to very special design situations (e. g. bearing capacity failure in the rubble mound).

Table 2. Results of the parameter study

load cases	Failure modes		
	bearing resistance failure in subsoil fm 1	limitation of eccentricity fm 2	sliding along the base of the caisson fm 4
wave crest lc 2	45.0 % (5,903,478)	15.0 % (2,224,422)	20.8 % (2,711,880)
wave trough lc 3	12.8 % (1,664,388)	4.02 % (525,384)	0.006% (720)

The results show that bearing resistance failure in the subsoil is the most important failure mode. It determines the caisson dimensions in over 50% of all investigated cases. This was not self-evident, because in traditional design practice it was usually considered only roughly by a certain threshold value of the stresses transmitted to the soil (Oumeraci et al., 2001). However, the limitation of the eccentricity and sliding along the caisson base cannot be neglected. Their importance depends on certain combinations of the input parameters of Table 1. The most relevant parameter is the eccentricity of the caisson weight e_C/B_C.

Figure 2 shows that for negative eccentricities (directed to harbour side) bearing resistance failure and the limitation of the eccentricity are the only relevant failure modes. Negative eccentricities cause an extreme increase of the bending moment acting on the structure in the case of wave crest and therefore strongly influence both failure modes. Which of them finally governs design depends on the shear strength of the subsoil φ'_s in relation to e_C/B_C.

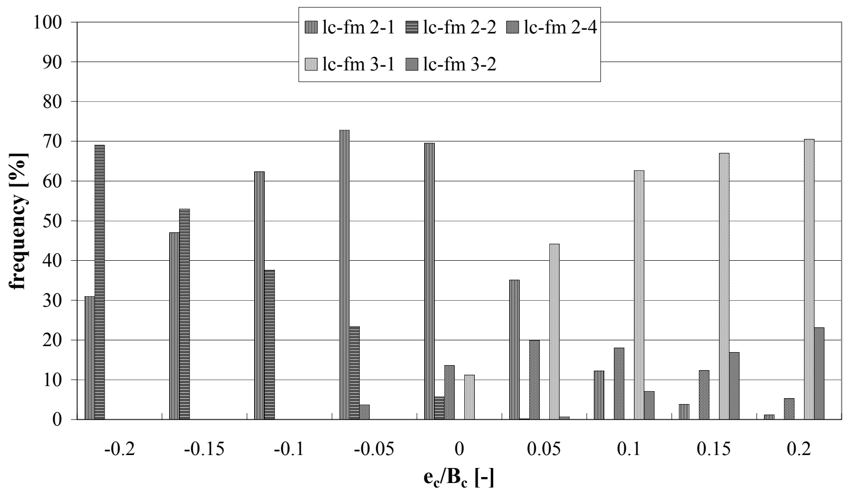

Figure 2. Relevant load case - failure mode combinations depending on e_C/B_C

For positive eccentricities each of the failure modes may be relevant. However, bearing resistance failure dominates especially in the case of a small shear strength of the subsoil ($\varphi'_s < 40°$). In Fig. 4 the influence of φ'_s is depicted for $e_C/B_C = 0.1$. Apparently sliding along the caisson base is relevant especially for a high shear strength of the subsoil ($\varphi'_s \geq 37.5°$). Figure 3 also shows the dominance of load case wave trough. For low wave heights the loading in case of wave trough is higher than the one in case of wave crest. Together with a positive eccentricity of the breakwater dead load this extremely increases the resulting bending moment acting on the structure.

Although several important interactions are evident the results finally show that none of the failure modes clearly dominates over the whole range of the input parameters. The reason is that the associated limit state equations are inconsistent and strongly correlated by their input parameters. Hence, splitting up the ULS of a shallow foundation into single limit states is artificial and cannot reflect the physical behaviour very well. As a consequence the evaluation of the actual safety of a shallow foundation is difficult.

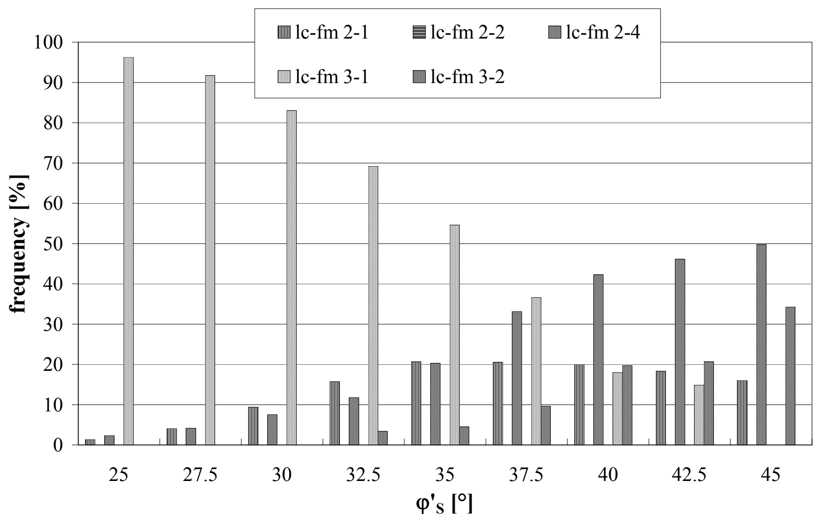

Figure 3. Relevant load case - failure mode combinations depending on φ'_s for $e_C/B_C = 0.1$

In Fig. 4 the bearing resistance and the sliding resistance of an example breakwater are depicted in the plane of vertical (F_1) and horizontal (F_2) loading for a fixed eccentricity of $M_3/(F_1 \cdot B_C) = 0.12$. The input parameters are taken from de Groot et al. (1996). With a fictitious load with a vertical component of $F_1 = 15\ \text{MN}$ and a horizontal component of $F_2 = 2.55\ \text{MN}$ the different design procedures can be explained.

When proving bearing resistance only the vertical components of loading and resistance are compared. This concept implies that the load path is always radial and the resistance is determined for this load path as well. When proving sliding resistance the horizontal load component is compared to the horizontal resistance depending on the actual vertical load component. This corresponds to a steplike load path. The distances between actual and admissible load components shown in Fig. 5 represent the actual safety level.

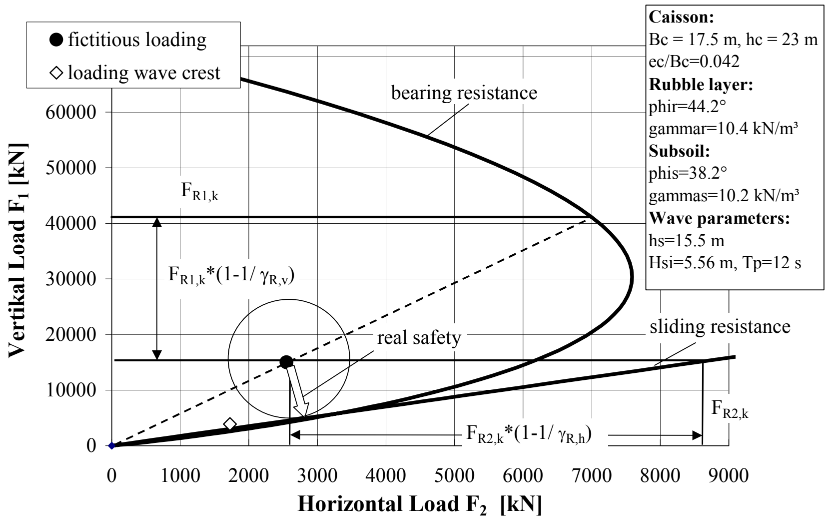

Figure 4. Interaction diagram for the stability analysis of an example breakwater

For the fictitious loading the calculated safety factors due to EC7 are $\gamma_{R,v} = 2.7 > 1.4$ for bearing resistance and $\gamma_{R,h} = 3.3 > 1.1$ for sliding resistance, indicating a sufficient safety. However, this design procedure does not reflect the real safety of the foundation, which is described by the closest distance of the actual loading to the resistance of the foundation as indicated by the arrow in Fig. 5 (Butterfield, 1993). Additional load components acting along this path are most hazardous for the foundation. Hence, for arbitrary load paths only additional load components acting within the circle sketched in Fig. 5 are admissible.

If we now consider the load case wave crest the consequences for the safety of the breakwater can be illustrated. With the wave parameters given in Fig. 4 the load components are $F_1 = 3.89\ \text{MN}$, $F_2 = 1.72\ \text{MN}$ and $M_3 = 7.78\ \text{MN}$. The load vector $\vec{Q}$ in the $F_1 - F_2 - \text{plane}$ is therefore:

$$\vec{Q} = \begin{bmatrix} 3.89 & 1.72 \end{bmatrix} \quad \text{with} \quad \left|\vec{Q}\right| = \sqrt{3.89^2 + 1.72^2} = 4.25\ \text{MN} \tag{1}$$

The safety factors acc. to EC7 are $\gamma_{R,v} = 3.1$ for bearing resistance and $\gamma_{R,h} = 1.3$ for sliding resistance. The closest distance to the resistance, which is the sliding resistance in this case, corresponds to a maximum additional load component of $\Delta\vec{Q} = 0.43\ \mathrm{MN}$ in the $F_1 - F_2 - \mathrm{plane}$.
Such a critical load situation may occur if the wave height is higher than assumed for design, resulting in an increase of the horizontal load but a decrease of the vertical load due to uplift forces.

Now, the actual safety of the structure can be calculated acc. to Butterfield (1993):

$$\gamma_R = \left(\vec{Q} + \Delta\vec{Q}\right) / \vec{Q} = (4.25 + 0.43)/4.25 = 1.10 \tag{2}$$

which is apparently less than the safety factors predicted with the current LSD. However, these factors are only valid for the load paths assumed in the design procedure and do not reflect the actual safety level. This limitation is problematic especially if the stability of a structure is influenced by parameters with a high uncertainty like the wave parameters in the example presented here.

Two main conclusions may be derived from the aforementioned illustration:

- First of all, the complex behaviour of the foundation should be defined without distinguishing different failure modes.
- Secondly, a safety concept is needed which takes into account the load path dependency.

In the following the concept of a new model which consistently describes the behaviour of a shallow foundation from initial loading up to failure is introduced.

Consistent design model for shallow foundations

The new model includes two components. The first component is a failure condition which describes the ULS of a shallow foundation without distinguishing different failure modes. The second component is a displacement rule which reflects the complete load-displacement relation before the system reaches its ultimate limit state. Thus, this component describes the SLS.

Failure Condition

For the general case a single footing is loaded by a vertical load F_1, horizontal load components F_2 and F_3, a torsional moment M_1 and bending moment components M_2 and M_3 (Fig. 5). The load components are summarized in the load vector $\vec{Q}$:

$$\vec{Q}^T = [F_1\, F_2\, F_3\, M_1\, M_2\, M_3] \tag{3}$$

For the basic case of a footing on non-cohesive soil without embedment the geometry of the footing described by the side ratio $\overline{b} = b_2/b_3$, weight γ, shear strength $\tan\varphi'$ of the soil and a quantity μ_S describing the roughness of the footing base have to be considered as well (Fig. 5).

With these input parameters the failure condition of the general form

$$F\left(\vec{Q}, \overline{b}, \gamma, \tan\varphi', \mu_S\right) = 0 \tag{4}$$

is defined by the following expression:

$$\sqrt{\frac{F_2^2 + F_3^2}{(a_1 \cdot F_{10})^2} + \frac{M_1^2}{(a_2 \cdot b_3 \cdot F_{10})^2} + \frac{M_2^2 + M_3^2}{(a_3 \cdot b_2 \cdot F_{10})^2}} - \frac{F_1}{F_{10}} \cdot \left(1 - \frac{F_1}{F_{10}}\right)^{\alpha} = 0 \tag{5}$$

In (5) quantity F_{10} is the resistance of a footing under pure vertical loading, which can be calculated using the traditional bearing resistance formula (e. g. acc. to EC7).

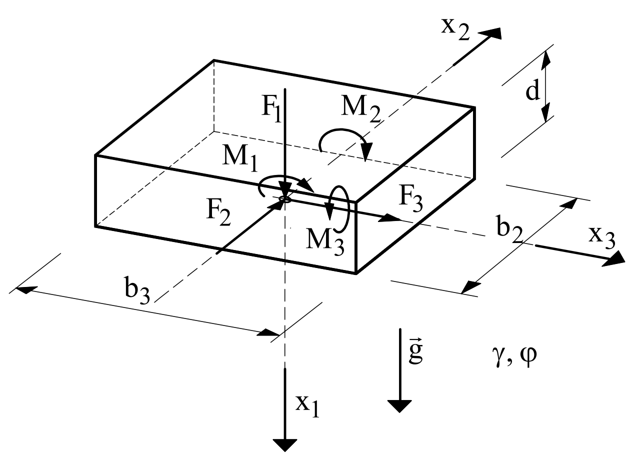

Figure 5. Geometry and loading

In an interaction diagram like the one in Fig. 5 the failure condition spans a failure surface, which is the outer boundary of the admissible loading. The parameters $a_{1,2,3}$ govern the inclination of this failure surface for small vertical loading where the limit states sliding and limitation of the eccentricity have previously been relevant (see Fig. 4). These limit states are integrated by defining the parameters $a_{1,2,3}$ and α acc. to (6).

$$a_1 = \pi/2 \cdot \mu_S \cdot \tan\varphi' \cdot e^{-\pi/3 \cdot \tan\varphi'}, \; a_2 = 0.098 \cdot \left(1+\overline{b}\right), \; a_3 = 0.42, \; \alpha = 1.3 \quad (6)$$

The limit state uplift is already included in (5), because only positive vertical loads are admissible. The parameters have been derived from an analysis of numerous small scale model tests. As an example Fig. 6 shows the failure condition for general loading compared to failure loads of some small scale model tests. More details are described in Lesny and Richwien (2002) or Lesny et al. (2002).

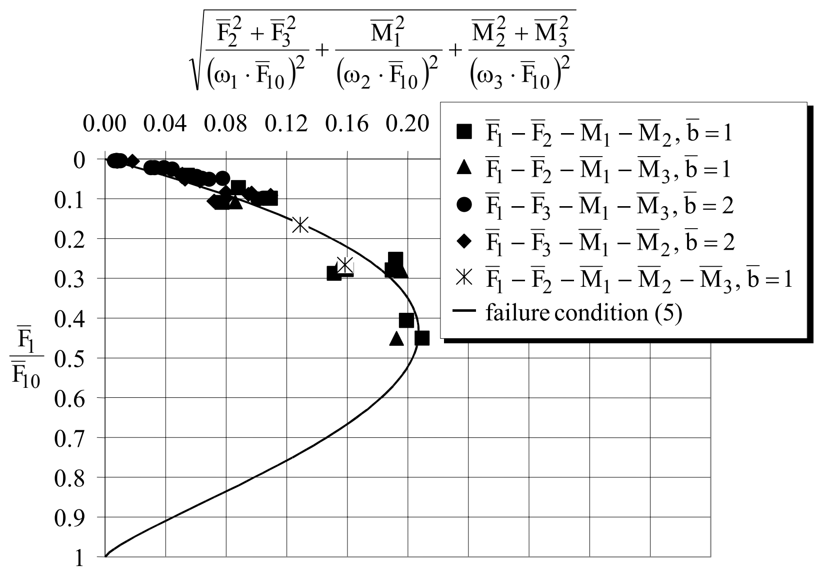

Figure 6. Failure condition for general loading vs. failure loads from small scale model tests (Lesny et al., 2002)

Displacement Rule

The displacements and rotations of the foundation due to arbitrary loading inside the failure surface are described by the displacement rule. The displacements u_i and rotations ω_i are summarized in a displacement vector:

$$\vec{u}^T = [u_1 \quad u_2 \quad u_3 \quad \omega_1 \quad \omega_2 \quad \omega_3]^T \tag{7}$$

Due to the complex interaction of load components, displacements and rotations the displacement rule is formulated using the well-known strain hardening plasticity theory with isotropic hardening (e. g. Zienkiewicz, 1988). Hence, displacements and rotations are calculated according to (8), assuming that all deformations are plastic.

$$d\vec{u} = \frac{1}{H} \cdot \left(\frac{\partial F}{\partial \vec{Q}} \right)^T \cdot \frac{\partial G}{\partial \vec{Q}} \cdot \Delta \vec{Q} \tag{8}$$

The components of the displacement rule are a yield surface described by the yield condition F:

$$F(\vec{Q}, \chi_a) = \sqrt{\frac{F_2^2 + F_3^2}{(a_1 \cdot F_{10})^2} + \frac{M_1^2}{(a_2 \cdot b_3 \cdot F_{10})^2} + \frac{M_2^2 + M_3^2}{(a_3 \cdot b_2 \cdot F_{10})^2}} - \frac{F_1}{F_{10}} \cdot \left(1 - \frac{F_1}{\chi_a \cdot F_{10}}\right)^{\alpha} = 0 \tag{9}$$

with the parameter $a_{1,2,3}$ of (5), a plastic potential G:

$$G(\vec{Q}, \chi_b) = \sqrt{\frac{F_2^2 + F_3^2}{(c_1 \cdot F_{10})^2} + \frac{M_1^2}{(c_2 \cdot b_3 \cdot F_{10})^2} + \frac{M_2^2 + M_3^2}{(c_3 \cdot b_2 \cdot F_{10})^2}} - \frac{F_1}{F_{10}} \cdot \left(1 - \frac{F_1}{\chi_b \cdot F_{10}}\right)^{\beta} = 0 \tag{10}$$

and a hardening function H:

$$H = -\frac{\partial F}{\partial \chi_a} \cdot \frac{\partial \chi_a}{\partial \vec{u}} \cdot \frac{\partial G}{\partial \vec{Q}} \tag{11}$$

The yield surface acc. to (9) expands due to isotropic hardening until the failure surface defined by (5) is reached. Thus, the parameters c_i and β in

(10) have to be determined as functions of a_i and α, respectively. The hardening parameter χ_a in (11) is formulated according to (12).

$$\chi_a = \chi_a(\vec{u}) = 1 - \exp\left(- \frac{A}{{F_{10}}^2} \cdot |u_1 \cdot F_{10}| \right) \qquad (12)$$

Many hardening laws (e. g. Nova et al, 1991) require small scale model tests under centric vertical loading to determine the factor A in (12). Since this is not convenient for practical applications, a procedure is under development to determine A from standard oedometer tests.

Figure 7 shows the results of the proposed model applied to the example breakwater of Fig. 4. Safety factors are not applied here. On the left side of Fig. 7 the failure condition and the loading in the $F_1 - F_2 -$plane and in the $F_1 - M_3/B_C -$plane are shown. Obviously, the stability of the breakwater is governed by the high horizontal loading. Only an increase in the vertical loading, i. e. of the caisson weight, would lead to a sufficient safety. The right side of Fig. 7 shows the vertical and horizontal displacements of the breakwater depending on the corresponding load components F_1 and F_2. However, due to some conservative assumptions made in the current version of the proposed model a caisson width of 21.0 m instead of 17.5 m was required to reach stability.

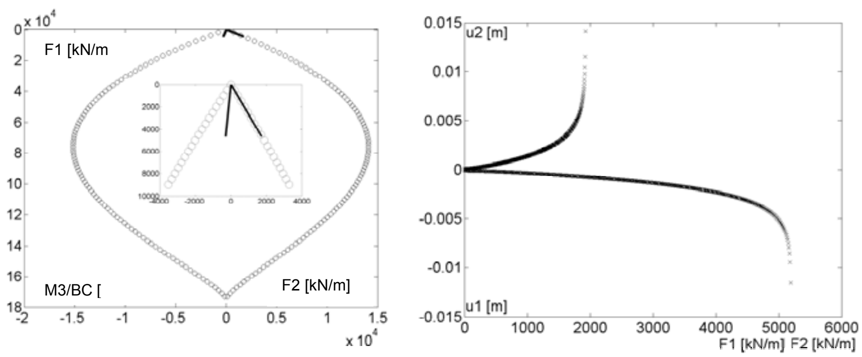

Figure 7. Failure condition (left) and load-displacement-curves (right) for the example breakwater

Conclusions

Based on the example of a vertical breakwater it was illustrated that the current limit state design may lead to an underestimation of the actual safety of a shallow foundation, because the load path dependency of the safety is not specifically considered. In the design of foundations under complex loading this is not apparent, because the stability of the foundation is governed by different limit states depending on the actual values of their input parameters.

A consistent design model was presented, which describes the complex behaviour of a shallow foundation under loading up to failure. The two components of this model, failure condition and corresponding displacement rule, consider both, ULS and SLS. Hence, the separate analysis of different limit states is no longer necessary. This model provides a distinct basis for the application of appropriate safety concepts which need to be implemented in the future.

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