Universität Duisburg-Essen
Computational Mechanics
Fakultät für Mathematik

Dipl.–Math. Andreas Fischle

Summer Term 2013
06/27/2013

Advanced Numerical Methods – Review Exercises.

Exercise 1:
Consider the initial value problems

- $y'(t) = (t-1)^2\, y(t), \quad y(0) = 1$
- $y'(t)\, y(t) = t, \qquad y(0) = 1$

1. Try to apply the Picard-Lindelöf existence and uniqueness theorem.
2. Solve the initial value problems.

Exercise 2:
Reduce the second order system of ODEs

$$y''(t) = Ay(t), \quad A \in \mathbb{R}^{n\times n}, \quad y(0) = y_0 \in \mathbb{R}^n, \quad y'(0) = 0 \in \mathbb{R}^n$$

to a first order system in the $2n$ auxiliary variables $z = [z_1, z_2]^T \in \mathbb{R}^{2n}$.
Write this in block matrix form and compute one step of the explicit Euler method for the reduced system.

Exercise 3:
Compute the set $\mathcal{A} \subset \mathbb{C}$ of absolute stability for the explicit and implicit Euler methods and draw them.

Exercise 4:
Write down the Runge-Kutta method corresponding to the following Butcher tableau:

$$\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1 & -1 & 2 & 0 \\ \hline & 1/6 & 2/3 & 1/6 \end{array}$$

Apply this scheme to the ODE

$$y'(t) = y(t), \quad y(0) = 1$$

and compute the first step.

Exercise 5:
Use the *method of lines* to discretize the advection boundary value problem

$$\begin{aligned}
\frac{\partial u}{\partial t}(x,t) + c\frac{\partial u}{\partial x}(x,t) &= 0, && (x,t) \in (0,2\pi)\times(0,\infty),\ c>0\\
u(x,0) &= \sin(x), && x \in [0,2\pi]\\
u(0,t) = u(2\pi,t) &= \sin(-ct), && t \in [0,\infty)
\end{aligned}$$

in space. Use the finite difference approximation

$$u'(x) = \frac{u(x+h)-u(x-h)}{2h} + O(h^2)$$

and an equidistantly spaced grid

$$0 = x_0 < x_1 < \ldots < x_n < x_{m+1} = 1$$

with step size $h = \frac{1}{m+1}$.

Exercise 6:
Let $u : \mathbb{R} \to \mathbb{R}$ be sufficiently smooth. Derive the finite difference approximation

$$u'(x) = \frac{u(x+h)-u(x-h)}{2h} + O(h^2).$$

Exercise 7:
Derive the modified Euler method from a suitable quadrature rule.

Exercise 8:
Prove that the order of consistency of Heun's method is 2.

Much success for your exam preparations.