Universität Duisburg-Essen
Computational Mechanics
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Advanced Numerical Methods – Homework 2.

Exercise 1
Consider the initial value problem (IVP) given by

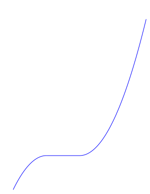
$$y' = 1 + y^2, \qquad y(0) = 0.$$

1. Does the theorem of Picard-Lindelöf apply here?
2. Compute an exact (analytic/closed form) solution to the IVP.
3. (MATLAB) Choose equidistant grid points $x_i, i = 0, \ldots, N$, with $x_0 = 0, x_N = 1$. Solve the ODE numerically by using Euler's *and* Heun's method on the same grid. Compare both numerical approximations with the exact solution in $x_N = 1$. Which method converges faster to the exact solution as you refine the mesh (i.e., for $h \to 0$)? Which numerical scheme takes less evaluations of the right hand side?

Exercise 2
Find all solutions to the ODE

$$y' = 2\sqrt{|y|}, \qquad y(0) = 0. \quad (*)$$

and prove that they solve $(*)$. Draw the direction field and some solutions. Discuss whether the theorem of Picard-Lindelöf applies here.
Hint: Consider the trajectory



Exercise 3
Look up Taylors theorem in 1D in the literature and write it down. Compute the following Taylorexpansions:

1. $f(x) = x^7 + 4x^2 - x + 3$ at $x_0 = 0$, up to terms of order 7
2. $g(x) = 1/x$ at $x_0 = 1$, up to terms of order 4
3. $h(x) = \sin(x)$ at $x_0 = \frac{\pi}{2}$, up to terms of order 6

Exercise 4
Starting from an ODE

$$y'(x) = f(x, y(x)), \quad f(x, y) \in C^2(\mathbb{R}^2, \mathbb{R})$$

derive the identities

$$y''(x) = f_x(x, y(x)) + f_y(x, y(x))f(x, y(x)) \tag{1}$$
$$y^{(3)}(x) = \dots \tag{2}$$

Due Date: 04/25/2013 .