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Computational Mechanics
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Advanced Numerical Methods – Homework 3.

Exercise 1

Write a MATLAB program to calculate the approximation to $y(0.3)$ for the IVP

$$y' = x^2 + y^2, \quad y(0) = 0;$$

Implement the methods of Euler and Heun. Use the approximation using a step length of 10^{-4} as a reference. What do you observe if you choose smaller and smaller step lengths, i.e. $h = 0.1, 0.05, 0.025 \ldots$.

Exercise 2

Consider the scalar IVP

$$y'(x) = f(x, y(x)), \quad f : \mathbb{R}^2 \to \mathbb{R} \tag{1}$$
$$y(0) = y_0 \; . \tag{2}$$

1. Write the IVP in an equivalent integral form, i.e., as an equivalent integral equation.
2. Try to derive the explicit Euler's and Heun's method by applying suitable quadrature rules for the approximation of the integral equation.
3. How does the scheme look like which is obtained from the midpoint rule?

Exercise 3

The following is a so-called **linear system of ODEs** with **constant coefficients**

$$\begin{pmatrix} u'(t) \\ v'(t) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} .$$

1. Find out (by studying the literature if necessary) how to solve such a system, assuming that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ can be diagonalized over the complex numbers! Write down the procedure.
2. Solve the system you obtain for the choice of matrix coefficients

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

3.* Solve the system you obtain for

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} .$$

Hint: The last matrix can be diagonalized over the complex numbers.

Due Date: 05/02/2013 .