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Computational Mechanics
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# Advanced Numerical Methods – Homework 4.

The following programming project is due in **two** weeks and you can earn 4 bonus points for the exam. Next week you will receive another set of homeworks.

**MATLAB Programming Project 1 (4 Bonus Points)**

Consider the following second-order system of ODEs modelling a particle at position $x(t) \in \mathbb{R}^2$ with mass $m$ moving in a force field $F : \mathbb{R}^2 \to \mathbb{R}^2$ (and obeying Newtonian physics):

$$F(x(t)) = m \cdot a(t).$$

Note that the acceleration vector $a(t)$ is the **second** derivative of the position vector $x(t)$ with respect to time, i.e.,

$$a(t) = \frac{\mathrm{d}}{\mathrm{d}t} v(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2} x(t),$$

where $v$ denotes the velocity of the particle.

At time $t = 0$ the particle is located at $x^{(0)} = (1, 0)^T$ and has the initital velocity $v^{(0)} = (0, 1/\sqrt{2})^T$. For the force field we assume $F(x_1, x_2) = (-x_2, x_1)^T$.

Introducing the components of the velocities as a new set of variables in the ODE, we can reduce this initial value problem to an equivalent first order system of ODEs

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} x_1(t) \\ x_2(t) \\ v_1(t) \\ v_2(t) \end{pmatrix} = \begin{pmatrix} v_1(t) \\ v_2(t) \\ F_1(x_1(t), x_2(t))/m \\ F_2(x_1(t), x_2(t))/m \end{pmatrix}.$$

If you prefer, you can also relabel $v_1 = x_3$ and $v_2 = x_4$.

1. Write down the update rule for the Euler, Modified Euler and Heun's scheme for the first order system given above.

2. Write a MATLAB program which simulates the particle with the Euler, Modified Euler and Heun's scheme, respectively, for the time interval $[0, T]$, for some $T > 0$ and with a given step size $h$.

3. Visualize the trajectory (as a curve in the plane) of the particle for each of the three methods in a plot. Take $T = 10[s]$ and use the three different step sizes $h = 10^{-1}, 10^{-2}$, $10^{-3}$. Do also produce superimposed plots showing the numerical approximations obtained from the different schemes but using the same stepsize.

4. Give a short comparison of the three different schemes you implemented based on your numerical results. Try to measure the global error of your approximations at $T = 10[s]$.

Hint: The exact solution to the initial value problem is given by:

$$
\begin{aligned}
x_1(t) &= \frac{1}{4} e^{-\frac{t}{\sqrt{2}}} \left( \left( 3e^{\sqrt{2}t} + 1 \right) \cos\left( \frac{t}{\sqrt{2}} \right) - \left( e^{\sqrt{2}t} + 1 \right) \sin\left( \frac{t}{\sqrt{2}} \right) \right), \\
x_2(t) &= \frac{1}{4} e^{-\frac{t}{\sqrt{2}}} \left( \left( 3e^{\sqrt{2}t} - 1 \right) \sin\left( \frac{t}{\sqrt{2}} \right) + \left( e^{\sqrt{2}t} - 1 \right) \cos\left( \frac{t}{\sqrt{2}} \right) \right)
\end{aligned}
$$

You can use this to check whether you're on the right track.

**Due Date: Programming: 05/17/2013 .**