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Advanced Numerical Methods – Homework 7.

Exercise 1:
Consider again the motion of a space probe exposed to the gravity of the moon and earth. We shall assume that the orbit can be described by two component functions $(u(t), v(t))^T$, i.e., that it is essentially planar. The coordinates are chosen such that the earth resides at $(0,0)^T$ and the moon is located at $(1,0)^T$ and obtain:

$$\ddot{u} = u + 2\dot{v} - (1-\mu)\frac{u+\mu}{[(u+\mu)^2+v^2]^{3/2}} - \mu\frac{u-1+\mu}{[(u-1+\mu)^2+v^2]^{3/2}},$$
$$\ddot{v} = v - 2\dot{u} - (1-\mu)\frac{v}{[(u+\mu)^2+v^2]^{3/2}} - \mu\frac{v}{[(u-1+\mu)^2+v^2]^{3/2}}\ .$$

Here, the parameter μ denotes the mass of the moon relative to the earth. Note that we used the dot notation to indicate time derivatives, i.e., $\dot{u} := \frac{\mathrm{d}u}{\mathrm{d}t}$. In Homework 5 you transformed the system of ordinary differential equations into a first order system in y_1, y_2, y_3, y_4. Consider the *implicit* Euler method for the corresponding reduced first order system, which is given by

$$y_{i+1} = y_i + hf(t_{i+1}, y_{i+1}).$$

1. Write down a fixed point iteration for y_{i+1}. Recall the conditions for the convergence of the fixed point iteration.
   (**Note:** You do not have to prove the convergence of the fixed point iteration).

2. Rewrite the implicit Euler scheme as a root finding problem $G(y_{i+1}) = 0$ where G is some appropriate auxiliary function. Work out Newtons method to compute a zero of G explicitly, i.e., write down the Newton iteration and the system to be solved in each Newton iteration.
   What might be a good initial value for the Newton iteration?