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Advanced Numerical Methods – Homework 8.

Exercise 1:

Use the MATLAB functions `ode23` and `ode45` to solve the IVP described on the last homework sheet. Use $\mu = \frac{1}{82.45}$ and initial values:

$$u_0 = 1.2,\ v_0 = 0,\ \dot{u}_0 = 0,\ \dot{v}_0 = -1.049357509830350.$$

Plot the trajectory of the space probe during the interval $[0, 10]$ with both schemes using `odephas2` (cf. the MATLAB documentation). Turn on the statistics for the solver using `odeset('stats', 'on')` and summarize the differences between the two sets of solver statistics. Try to explain the statistics from the perspective of embedded Runge-Kutta methods.

Exercise 2:

Consider again the previous simulation of the space probe. Implement the implicit Euler method for this IVP. Use a Newton iteration scheme to compute the updates. Stop the Newton iteration if the residual norm satisfies `norm(r,2 ) < 1e-8` and make a plot which shows the number of Newton iterations computed for every timestep. Try different stepsizes, e.g, $h = 10^{-k}$, $k = 1, 2, 3$.

Exercise 3:

We want to simulate a harmonic oscillator with and without damping. The equations of movement are governed by Hooke's Law $F(x) = -kx$, where $x(t)$ denotes the displacement at time t and $k > 0$ is a spring constant. In the undamped case, we have the IVP

$$\begin{aligned} x(0) &= 0 \\ x'(t) &= 1 \\ mx''(t) &= -kx(t). \end{aligned}$$

A *damped* harmonic oscillator is subject to an additional friction force proportional to the velocity x'. This gives a total force $F(x) = -kx - bx'$, where $b > 0$ is the friction coefficient leading us to:

$$\begin{aligned} x(0) &= 0 \\ x'(t) &= 1 \\ mx''(t) &= -kx(t) - bx'(t). \end{aligned}$$

1. Write a MATLAB code which solves the *undamped* oscillator IVP with $m = 1$ and $k = 1$ numerically using the following schemes:
   (a) Explicit Euler
   (b) Heun's scheme
   (c) Classical Runge Kutta (4th order)

2. Do the same for the *damped* harmonic oscillator, setting the friction coefficient to $b = 1/2$. You do not need to plot the exact solution.

Exercise 4:

Consider a point mass $p \in \mathbb{R}^2$ with unit mass $m = 1$ which is connected to the origin $(0, 0)$ by a spring with spring constant $k = 1$. Let $p(0) = (1, 0)^T$ and $p'(0) = (0, 1)^T$. The spring is *initially at rest*, i.e., it does not generate any forces in the initial configuration at $t = 0$. We turn off gravity and neglect friction.

1. Write down the equations of movement for the point mass p using Hooke's Law (without damping).
2. Use the matlab solvers `ode23` and `ode45` to solve and plot the trajectory of $p(t)$ during the time interval $[0, 100]$. Turn on the statistics by using `odeset('stats', 'on')`.

Hint: Reduce the respective IVPs to first order systems of differential equations first.