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Computational Mechanics
Fakultät für Mathematik

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Summer Term 2013
06/20/2013

Advanced Numerical Methods – Homework 9.

Exercise 1:
Plot the regions of absolute stability $\mathcal{A}$ for the schemes

1. Explicit Euler
2. Implicit Euler
3. Modified Euler
4. Heun scheme

using rejection sampling. Use the `rand` function in MATLAB to sample the square $[-2,2]+i[-2,2] \subset \mathbb{C}$ uniformly. If a random point is inside the set of absolute stability $\mathcal{A}$ it is displayed, otherwise it is rejected. This generates a plot of the set $\mathcal{A} \cap [-2,2]+i[-2,2] \subset \mathbb{C}$.

Exercise 2:
The solution to the *test equation*

$$z'(t) = \lambda z(t), \quad z(0) = 1, \quad t \in [0, \infty)$$

with $\lambda \in \mathbb{C}$ is given by $z(t) = \exp(t\lambda)$.

1. How does $|z(t)|$ depend on the real and imaginary parts of λ?
2. Determine the sets of values for $\lambda \in \mathbb{C}$ such that
   (a) $\lim_{t\to\infty} |z(t)| = 0$
   (b) $\lim_{t\to\infty} |z(t)| = \infty$
   (c) $\lim_{t\to\infty} |z(t)| = 1$

   and draw a picture of the corresponding regions in the complex plane.
3. What is the connection between the test equation with the asymptotic study of a given linear system of ODEs $y'(t) = Ky(t)$, assuming $K \in \mathbb{R}^{n\times n}$ can be diagonalized?

Exercise 3:

1. Consider the iteration scheme

$$z_{n+1} = \alpha z_n, \quad \alpha, z_0 \in \mathbb{C}.$$

   Find conditions for α such that

   (a) $\lim_{n\to\infty} |z_n| = 0$

   (b) $\lim_{n\to\infty} |z_n| = \infty$

   (c) $\lim_{n\to\infty} |z_n| = |z_0|$

2. Apply the explicit Euler scheme to the test equation. This gives an iterative scheme as described above. Compute the corresponding set $\mathcal{A}_h = \{\lambda \in \mathbb{C} : \lim_{n\to\infty} z_n = 0\}$.

3. Verify that $\lambda \in \mathcal{A}_h \iff z = \lambda h \in \mathcal{A}$.

4. Consider the sequence $(z_n)_{n\in\mathbb{N}}$ which is associated to the explicit Euler scheme. Compare the asymptotic behavior with the exact solution $z(t)$ for $\lambda \in \mathcal{A}_h$. What can you say about the asymptotic behavior of the explicit Euler scheme for $\lambda \notin \mathcal{A}_h$? What happens with the set $\mathcal{A}_h$ as $h \to 0$? What can you conclude?

Please turn in until: 06/27/2013