

Example regarding the inverse iteration

$$A = \begin{pmatrix} 10 & 1 \\ 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = 10, \lambda_2 = 1 \Rightarrow x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ -9 \end{pmatrix}$$

Choose: $\mu = 0$ and $q^{(0)} = (1, 1)^T$, $q^{(0)} = (\frac{1}{2}, \frac{1}{2})^T$, or $q^{(0)} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})^T$

	$\ \cdot\ _\infty$	$\ \cdot\ _1$	$\ \cdot\ _2$
	$x_2 = \begin{pmatrix} \frac{1}{9} \\ -1 \end{pmatrix} \approx \begin{pmatrix} 0.111 \\ -1 \end{pmatrix}$	$x_2 = \begin{pmatrix} \frac{1}{10} \\ -\frac{9}{10} \end{pmatrix} = \begin{pmatrix} 0.1 \\ -0.9 \end{pmatrix}$	$x_2 = \frac{1}{\sqrt{82}} \begin{pmatrix} 1 \\ -9 \end{pmatrix} \approx \begin{pmatrix} 0.110 \\ -0.994 \end{pmatrix}$
$Az^{(1)} = q^{(0)} \Leftrightarrow z^{(1)} =$	$\begin{pmatrix} \frac{1-1}{10} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} \frac{\frac{1}{2}-\frac{1}{2}}{\frac{1}{10}} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} \frac{1-1}{\sqrt{2}\cdot 10} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$\ z^{(1)}\ $	1	$0 + \frac{1}{2} = \frac{1}{2}$	$\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$
$q^{(1)}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \cdot (\frac{1}{\sqrt{2}})^{-1} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$\nu^{(1)}$ $= (q^{(1)})^T A q^{(1)}$	$(0, 1) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1$	$(0, 1) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1$	$(0, 1) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1$
$Az^{(2)} = q^{(1)} \Leftrightarrow z^{(2)} =$	$\begin{pmatrix} -\frac{1}{10} \\ 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{10} \\ 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{10} \\ 1 \end{pmatrix}$
$\ z^{(2)}\ $	1	$\frac{1}{10} + 1 = \frac{11}{10}$	$\sqrt{\frac{1}{100} + 1} = \sqrt{\frac{101}{100}} = \frac{\sqrt{101}}{10}$

Example regarding the inverse iteration

$$A = \begin{pmatrix} 10 & 1 \\ 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = 10, \lambda_2 = 1 \Rightarrow x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ -9 \end{pmatrix}$$

	$\ \cdot\ _\infty$	$\ \cdot\ _1$	$\ \cdot\ _2$
$q^{(2)}$	$\begin{pmatrix} \frac{-1}{10} \\ 1 \end{pmatrix}$	$\begin{pmatrix} \frac{-1}{10} \cdot (\frac{11}{10})^{-1} \\ 1 \cdot (\frac{11}{10})^{-1} \end{pmatrix} = \begin{pmatrix} \frac{-1}{11} \\ \frac{10}{11} \end{pmatrix}$	$\begin{pmatrix} \frac{-1}{10} \cdot (\frac{\sqrt{101}}{10})^{-1} \\ 1 \cdot (\frac{\sqrt{101}}{10})^{-1} \end{pmatrix} = \frac{1}{\sqrt{101}} \begin{pmatrix} -1 \\ 10 \end{pmatrix}$
$\nu^{(2)}$ $= (q^{(2)})^T A q^{(2)}$	$(\frac{-1}{10}, 1) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$	$(\frac{-1}{11}, \frac{10}{11}) \begin{pmatrix} 0 \\ \frac{10}{11} \end{pmatrix} = \frac{100}{121} \approx 0.827$	$\frac{1}{101} (-1, 10) \begin{pmatrix} 0 \\ 10 \end{pmatrix} = \frac{100}{101} \approx 0.990$
$Az^{(3)} = q^{(2)} \Leftrightarrow z^{(3)} =$	$\begin{pmatrix} \frac{-11}{100} \\ 1 \end{pmatrix}$	$\begin{pmatrix} \frac{-1}{10} \\ \frac{10}{11} \end{pmatrix}$	$\frac{1}{\sqrt{101}} \begin{pmatrix} \frac{-11}{10} \\ 10 \end{pmatrix}$
$\ z^{(3)}\ $	1	$\frac{1}{10} + \frac{10}{11} = \frac{111}{110}$	$\sqrt{\frac{1}{101} (\frac{121}{100} + 100)} = \sqrt{\frac{10121}{10100}}$
$q^{(3)}$	$\begin{pmatrix} \frac{-11}{100} \\ 1 \end{pmatrix} = \begin{pmatrix} -0.110 \\ 1 \end{pmatrix}$	$\begin{pmatrix} \frac{-1}{10} \cdot (\frac{111}{110})^{-1} \\ \frac{10}{11} \cdot (\frac{111}{110})^{-1} \end{pmatrix} = \begin{pmatrix} \frac{-11}{111} \\ \frac{100}{111} \end{pmatrix} \approx \begin{pmatrix} -0.099 \\ 0.901 \end{pmatrix}$	$\sqrt{\frac{10121}{10100 \cdot 101}} \begin{pmatrix} \frac{-11}{10} \\ 10 \end{pmatrix} = \sqrt{\frac{10121}{1020100}} \begin{pmatrix} \frac{-11}{10} \\ 10 \end{pmatrix} \approx \begin{pmatrix} 0.110 \\ 0.996 \end{pmatrix}$
$\nu^{(3)}$ $= (q^{(3)})^T A q^{(3)}$	$(\frac{-11}{100}, 1) \begin{pmatrix} \frac{-1}{10} \\ 1 \end{pmatrix} = \frac{11}{1000} + 1 = \frac{1011}{1000} = 1.011$	$(\frac{-11}{111}, \frac{100}{111}) \begin{pmatrix} \frac{-10}{111} \\ \frac{100}{111} \end{pmatrix} = \frac{110}{111^2} + \frac{10000}{111^2} = \frac{10110}{12321} = \frac{3370}{4107} \approx 0.821$	$\frac{10121}{1020100} (\frac{-11}{10}, 10) \begin{pmatrix} -1 \\ 10 \end{pmatrix} = \frac{10121}{1020100} \frac{1011}{10} = \frac{10232331}{10201000} \approx 1.003$