Example regarding the power iteration

$$A = \begin{pmatrix} 10 & 1 \\ 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = 10, \lambda_2 = 1 \Rightarrow x_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ -9 \end{pmatrix}$$

Choose: $q^{(0)} = (1,1)^T$, $q^{(0)} = (\frac{1}{2}, \frac{1}{2})^T$, or $q^{(0)} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})^T$

	$\|\cdot\|_\infty$	$\|\cdot\|_1$	$\|\cdot\|_2$
$z^{(1)} = Aq^{(0)}$	$\begin{pmatrix} 10+1 \\ 1 \end{pmatrix} = \begin{pmatrix} 11 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 5+\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{11}{2} \\ \frac{1}{2} \end{pmatrix}$	$\frac{1}{\sqrt{2}} \begin{pmatrix} 10+1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 11 \\ 1 \end{pmatrix}$
$\|z^{(1)}\|$	11	$\frac{11+1}{2} = 6$	$\sqrt{\frac{121}{2} + \frac{1}{2}} = \sqrt{\frac{122}{2}} = \sqrt{61}$
$q^{(1)}$	$\begin{pmatrix} \frac{11}{11} \\ \frac{1}{11} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{11} \end{pmatrix}$	$\begin{pmatrix} \frac{11}{12} \\ \frac{1}{12} \end{pmatrix}$	$\begin{pmatrix} \frac{11}{\sqrt{2}\cdot\sqrt{61}} \\ \frac{1}{\sqrt{2}\cdot\sqrt{61}} \end{pmatrix} = \frac{1}{\sqrt{122}} \begin{pmatrix} 11 \\ 1 \end{pmatrix}$
$\nu^{(1)}$	$(1, \frac{1}{11}) \begin{pmatrix} \frac{111}{11} \\ \frac{1}{11} \end{pmatrix}$	$(\frac{11}{12}, \frac{1}{12}) \begin{pmatrix} \frac{111}{12} \\ \frac{1}{12} \end{pmatrix}$	$\frac{1}{122}(11,1) \begin{pmatrix} 111 \\ 1 \end{pmatrix}$
$= (q^{(1)})^T \underbrace{Aq^{(1)}}_{=z^{(2)}}$	$= \frac{111}{11} + \frac{1}{121} = \frac{1222}{121} \approx 10.099$	$= \frac{1221}{144} + \frac{1}{144} = \frac{1222}{144} = \frac{611}{72} \approx 8.486$	$= \frac{1}{122}(1221+1) = \frac{1222}{122} = \frac{611}{61} \approx 10.016$
$z^{(2)} = Aq^{(1)}$	$\begin{pmatrix} \frac{111}{11} \\ \frac{1}{11} \end{pmatrix}$	$\begin{pmatrix} \frac{111}{12} \\ \frac{1}{12} \end{pmatrix}$	$\frac{1}{\sqrt{122}} \begin{pmatrix} 111 \\ 1 \end{pmatrix}$
$\|z^{(2)}\|$	$\frac{111}{11}$	$\frac{111}{12} + \frac{1}{12} = \frac{112}{12} = \frac{28}{3}$	$\sqrt{\frac{12221}{122} + \frac{1}{122}} = \sqrt{\frac{12322}{122}} = \sqrt{101}$
$q^{(2)}$	$\begin{pmatrix} \frac{111}{11} \cdot (\frac{111}{11})^{-1} \\ \frac{1}{11} \cdot (\frac{111}{11})^{-1} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{111} \end{pmatrix}$	$\begin{pmatrix} \frac{111}{12} \cdot (\frac{112}{12})^{-1} \\ \frac{1}{12} \cdot (\frac{112}{12})^{-1} \end{pmatrix} = \begin{pmatrix} \frac{111}{112} \\ \frac{1}{112} \end{pmatrix}$	$\begin{pmatrix} \frac{111}{\sqrt{122}\cdot\sqrt{101}} \\ \frac{1}{\sqrt{122}\cdot\sqrt{101}} \end{pmatrix} = \frac{1}{\sqrt{12322}} \begin{pmatrix} 111 \\ 1 \end{pmatrix}$

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	$\Vert \cdot \Vert_\infty$	$\Vert \cdot \Vert_1$	$\Vert \cdot \Vert_2$
$\nu^{(2)}$	$(1, \frac{1}{111}) \begin{pmatrix} \frac{1111}{111} \\ \frac{1}{111} \end{pmatrix}$	$(\frac{111}{112}, \frac{1}{112}) \begin{pmatrix} \frac{1111}{112} \\ \frac{1}{112} \end{pmatrix}$	$\frac{1}{12322}(111, 1) \begin{pmatrix} 1111 \\ 1 \end{pmatrix}$
$= (q^{(2)})^T \underbrace{Aq^{(2)}}_{=z^{(3)}}$	$= \frac{1111}{111} + \frac{1}{12321} = \frac{123322}{12321} \approx 10.009$	$= \frac{123321}{112^2} + \frac{1}{112^2} = \frac{61661}{6272} \approx 9.8312$	$= \frac{123322}{12322} = \frac{61661}{6161} \approx 10.008$
$z^{(3)} = Aq^{(2)}$	$\begin{pmatrix} \frac{1111}{111} \\ \frac{1}{111} \end{pmatrix}$	$\begin{pmatrix} \frac{1111}{112} \\ \frac{1}{112} \end{pmatrix}$	$\frac{1}{\sqrt{12322}} \begin{pmatrix} 1111 \\ 1 \end{pmatrix}$
$\Vert z^{(3)} \Vert$	$\frac{1111}{111}$	$\frac{1111}{112} + \frac{1}{112} = \frac{1112}{112} = \frac{136}{14}$	$\sqrt{\frac{1234321}{12322} + \frac{1}{12322}} = \sqrt{\frac{617161}{6161}}$
$q^{(3)}$	$\begin{pmatrix} \frac{1111}{111} \cdot (\frac{1111}{111})^{-1} \\ \frac{1}{111} \cdot (\frac{1111}{111})^{-1} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{1111} \end{pmatrix}$ $\approx \begin{pmatrix} 1 \\ 0.001 \end{pmatrix}$	$\begin{pmatrix} \frac{1111}{112} \cdot (\frac{1112}{112})^{-1} \\ \frac{1}{112} \cdot (\frac{1112}{112})^{-1} \end{pmatrix} = \begin{pmatrix} \frac{1111}{1112} \\ \frac{1}{1112} \end{pmatrix}$ $\approx \begin{pmatrix} 0.999 \\ 0.001 \end{pmatrix}$	$\sqrt{\frac{6161}{617161 \cdot 12322}} \begin{pmatrix} 1111 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{1234322}} \begin{pmatrix} 1111 \\ 1 \end{pmatrix}$ $\approx \begin{pmatrix} 1 \\ 0.001 \end{pmatrix}$
$\nu^{(3)}$	$(1, \frac{1}{1111}) \begin{pmatrix} \frac{11111}{1111} \\ \frac{1}{1111} \end{pmatrix}$	$(\frac{1111}{1112}, \frac{1}{1112}) \begin{pmatrix} \frac{11111}{1112} \\ \frac{1}{1112} \end{pmatrix}$	$\frac{1}{1234322}(1111, 1) \begin{pmatrix} 11111 \\ 1 \end{pmatrix}$
$= (q^{(3)})^T A q^{(3)}$	$= \frac{11111}{1111} + \frac{1}{1111^2} = \frac{12344322}{1234321} \approx 10.001$	$= \frac{12344321}{1112^2} + \frac{1}{1112^2} = \frac{6172161}{618272} \approx 9.983$	$= \frac{12344322}{1234322} = \frac{6172161}{617161} \approx 10.001$