

Introduction to Numerical Methods Tutorial 11

(*)-Exercise 1: ($4 \times 4 = 16$ points)

Test the 4 methods introduced in the lecture for the one dimensional case to solve nonlinear equations. Therefore always make the following steps

- Formulate the problem as root finding or fixed point problem.
- Plot the problem, i.e., plot the function(s) to see where the root or fixed point is approximately located.
- Choose start iterates and compute 5 steps with an accuracy of at least 5 digits, i.e., try to work with exact arithmetic as long as possible round only if the number seems to have an infinite number of digits. Don't forget to have a look how good your solution already is.

Use all 4 methods, i.e., choose a different method for each problem. You can choose them as you like the methods suggested in the brackets are only suggestions.

- (i) $x^2 - 2 = 0$ for $x \in \mathbb{R}_+$, i.e., only positive values. (Newton method)
- (ii) Find the first positive $x \neq 0$ such that $\tan(x) = \tanh(x)$, with $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. (Bisection method)
- (iii) $\arctan(x) = x$ for $x \in (\frac{\pi}{2}, \frac{3\pi}{2})$ (Fixed point iteration)
Hint: Take care which values your pocket calculator or computer gives for arctan you may have to change the functions a little bit.
- (iv) $x^3 = 2$ (Secant method)

(**)-Exercise 2: ($(5 + 5) + (2 + 2) = 14$ points)

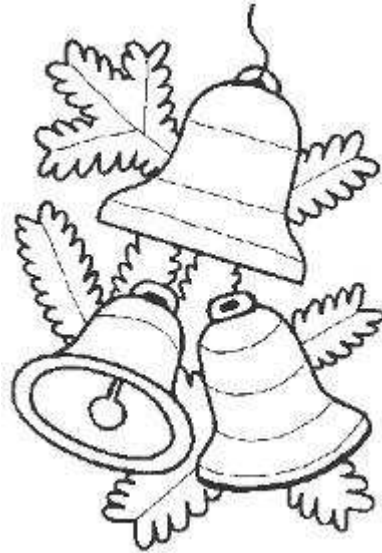
- (i) Write two programs:
 - (1) for the Bisection method to solve $x^2 - 2 = 0$ for $x \in \mathbb{R}_+$ and with variable initial values a and b ,

- (2) for the Fixed point iteration to solve $\arctan(x) = x$ for $x \in (\frac{\pi}{2}, \frac{3\pi}{2})$ with variable initial guess $x^{(0)}$, do not forget what you did in Exercise 1.
- (ii) Add a graphical output to the programs in part (i) to illustrate the computing process.

The points for the (**)-exercise are additional points. This means you can earn points which are not added to the overall amount of points of the (*)-exercises.

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Seasons greeting to all of you.



Have nice Christmas holidays
and a happy New Year!