

Solution of Tutorial 14 Exercise 2 (ii) and 3 (ii)

$$A = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = \frac{3 + \sqrt{5}}{2} \approx 2.618, \lambda_{2,3} = 1, \lambda_4 = \frac{3 - \sqrt{5}}{2} \approx 0.382, \Rightarrow x_1 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5} - 1 \\ 0 \end{pmatrix}, x_4 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5} + 1 \\ 0 \end{pmatrix}$$

Choose: $\mu = 0$ and $q^{(0)} = (0, 0, 1, 0)^T$

	$\ \cdot\ _\infty$	$\ \cdot\ _1$	$\ \cdot\ _2$
x_2	$\begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 0 \\ 1 \\ 0 \end{pmatrix} \approx \begin{pmatrix} -0.618 \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{5}-3}{2} \\ 0 \\ \frac{\sqrt{5}-1}{2} \\ 0 \end{pmatrix} \approx \begin{pmatrix} -0.382 \\ 0 \\ 0.618 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\sqrt{\frac{2}{5+\sqrt{5}}} \\ 0 \\ \frac{1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}} \\ 0 \end{pmatrix} \approx \begin{pmatrix} -0.526 \\ 0 \\ 0.851 \\ 0 \end{pmatrix}$
$Az^{(1)} = q^{(0)} \Leftrightarrow z^{(1)} =$	$\begin{pmatrix} 0-1 \\ 0 \\ 2 \cdot 1-0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0-1 \\ 0 \\ 2 \cdot 1-0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0-1 \\ 0 \\ 2 \cdot 1-0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$
$\ z^{(1)}\ $	2	$1 + 2 = 3$	$\sqrt{1+4} = \sqrt{5}$
$q^{(1)}$	$\begin{pmatrix} -\frac{1}{2} \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{3} \\ 0 \\ \frac{2}{3} \\ 0 \end{pmatrix}$	$\frac{1}{\sqrt{5}} \begin{pmatrix} -1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$
$\nu^{(1)}$	$(-\frac{1}{2}, 0, 1, 0) \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \\ 0 \end{pmatrix}$	$(-\frac{1}{3}, 0, \frac{2}{3}, 0) \begin{pmatrix} 0 \\ 0 \\ \frac{1}{3} \\ 0 \end{pmatrix}$	$\frac{1}{5}(-1, 0, 2, 0) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$
$= (q^{(1)})^T A q^{(1)}$	$= \frac{1}{2}$	$= \frac{2}{9} \approx 0.222$	$= \frac{2}{5} = 0.4$
$Az^{(2)} = q^{(1)} \Leftrightarrow z^{(2)} =$	$\begin{pmatrix} -\frac{1}{2}-1 \\ 0 \\ 2 \cdot 1 - (-\frac{1}{2}) \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ 0 \\ \frac{5}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{3}-\frac{2}{3} \\ 0 \\ 2 \cdot \frac{2}{3} - (-\frac{1}{3}) \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{5}{3} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{\sqrt{5}} - \frac{2}{\sqrt{5}} \\ 0 \\ 2 \cdot \frac{2}{\sqrt{5}} - (-\frac{1}{\sqrt{5}}) \\ 0 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} -3 \\ 0 \\ 5 \\ 0 \end{pmatrix}$

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$$A = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = \frac{3 + \sqrt{5}}{2} \approx 2.618, \lambda_{2,3} = 1, \lambda_4 = \frac{3 - \sqrt{5}}{2} \approx 0.382, \Rightarrow x_1 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5} - 1 \\ 0 \end{pmatrix}, x_4 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5} + 1 \\ 0 \end{pmatrix}$$

	$\ \cdot\ _\infty$	$\ \cdot\ _1$	$\ \cdot\ _2$
$\ z^{(2)}\ $	$\frac{5}{2}$	$1 + \frac{5}{3} = \frac{8}{3}$	$\sqrt{\frac{9}{5} + 5} = \sqrt{\frac{34}{5}}$
$q^{(2)}$	$\begin{pmatrix} -\frac{3}{5} \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.6 \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{3}{8} \\ 0 \\ \frac{5}{8} \\ 0 \end{pmatrix} = \begin{pmatrix} -0.375 \\ 0 \\ 0.625 \\ 0 \end{pmatrix}$	$\frac{\sqrt{5}}{\sqrt{34 \cdot 5}} \begin{pmatrix} -3 \\ 0 \\ 5 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{34}} \begin{pmatrix} -3 \\ 0 \\ 5 \\ 0 \end{pmatrix} \approx \begin{pmatrix} -0.515 \\ 0 \\ 0.858 \\ 0 \end{pmatrix}$
$\nu^{(2)}$	$(-\frac{3}{5}, 0, 1, 0) \begin{pmatrix} -\frac{1}{5} \\ 0 \\ \frac{2}{5} \\ 0 \end{pmatrix}$	$(-\frac{3}{8}, 0, \frac{5}{8}, 0) \begin{pmatrix} -\frac{1}{8} \\ 0 \\ \frac{1}{4} \\ 0 \end{pmatrix}$	$\frac{1}{34}(-3, 0, 5, 0) \begin{pmatrix} -1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$
$= (q^{(2)})^T A q^{(2)}$	$= \frac{3}{25} + \frac{2}{5} = \frac{13}{25} = 0.52$	$= \frac{3}{64} + \frac{5}{32} = \frac{13}{64} \approx 0.203$	$= \frac{3+10}{34} = \frac{13}{34} \approx 0.382$
$Az^{(3)} = q^{(2)} \Leftrightarrow z^{(3)} =$	$\begin{pmatrix} -\frac{3}{5} - 1 \\ 0 \\ 2 \cdot 1 - (-\frac{3}{5}) \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{8}{5} \\ 0 \\ \frac{13}{5} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{3}{8} - \frac{5}{8} \\ 0 \\ 2 \cdot \frac{5}{8} - (-\frac{3}{8}) \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{13}{8} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{3}{\sqrt{34}} - \frac{5}{\sqrt{34}} \\ 0 \\ 2 \cdot \frac{5}{\sqrt{34}} - (-\frac{3}{\sqrt{34}}) \\ 0 \end{pmatrix} = \frac{1}{\sqrt{34}} \begin{pmatrix} -8 \\ 0 \\ 13 \\ 0 \end{pmatrix}$
$\ z^{(3)}\ $	$\frac{13}{5}$	$1 + \frac{13}{8} = \frac{21}{8}$	$\sqrt{\frac{64}{34} + \frac{169}{34}} = \sqrt{\frac{233}{34}}$
$q^{(3)}$	$\begin{pmatrix} -\frac{8}{13} \\ 0 \\ 1 \\ 0 \end{pmatrix} \approx \begin{pmatrix} -0.615 \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{8}{21} \\ 0 \\ \frac{13}{21} \\ 0 \end{pmatrix} \approx \begin{pmatrix} -0.381 \\ 0 \\ 0.619 \\ 0 \end{pmatrix}$	$\frac{\sqrt{34}}{\sqrt{233 \cdot 34}} \begin{pmatrix} -8 \\ 0 \\ 13 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{233}} \begin{pmatrix} -8 \\ 0 \\ 13 \\ 0 \end{pmatrix} \approx \begin{pmatrix} -0.524 \\ 0 \\ 0.852 \\ 0 \end{pmatrix}$

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	$\ \cdot\ _\infty$	$\ \cdot\ _1$	$\ \cdot\ _2$
$\nu^{(3)}$	$(-\frac{8}{13}, 0, 1, 0) \begin{pmatrix} -\frac{3}{13} \\ 0 \\ \frac{5}{13} \\ 0 \end{pmatrix}$	$(-\frac{8}{21}, 0, \frac{13}{21}, 0) \begin{pmatrix} -\frac{3}{21} \\ 0 \\ \frac{5}{21} \\ 0 \end{pmatrix}$	$\frac{1}{233}(-8, 0, 13, 0) \begin{pmatrix} -3 \\ 0 \\ 5 \\ 0 \end{pmatrix}$
$= (q^{(2)})^T A q^{(2)}$	$= \frac{24}{169} + \frac{5}{13} = \frac{89}{13} \approx \mathbf{0.527}$	$= \frac{24}{441} + \frac{65}{441} = \frac{89}{441} \approx \mathbf{0.202}$	$= \frac{24+64}{233} = \frac{89}{233} \approx \mathbf{0.382}$