Solution of Tutorial 14 Exercise 2 (i) and 3 (i)

$$A = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = \frac{3+\sqrt{5}}{2} \approx 2.618, \lambda_{2,3} = 1, \lambda_4 = \frac{3-\sqrt{5}}{2} \approx 0.382, \Rightarrow x_1 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5}-1 \\ 0 \end{pmatrix}, x_4 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5}+1 \\ 0 \end{pmatrix}$$

Choose: $q^{(0)} = (0,0,1,0)^T$

	$\|\cdot\|_\infty$		$\|\cdot\|_1$		$\|\cdot\|_2$	
x_1	$\begin{pmatrix} 1 \\ 0 \\ \frac{\sqrt{5}-1}{2} \\ 0 \end{pmatrix} \approx$	$\begin{pmatrix} 1 \\ 0 \\ 0.618 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{5}-1}{2} \\ 0 \\ \frac{3-\sqrt{5}}{2} \\ 0 \end{pmatrix} \approx$	$\begin{pmatrix} 0.618 \\ 0 \\ 0.382 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \sqrt{\frac{2}{5-\sqrt{5}}} \\ 0 \\ \frac{\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}} \\ 0 \end{pmatrix} \approx$	$\begin{pmatrix} 0.851 \\ 0 \\ 0.526 \\ 0 \end{pmatrix}$
$z^{(1)} = Aq^{(0)}$	$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$		$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$		$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$	
$\|z^{(1)}\|$	1		$1+1=2$		$\sqrt{1+1}=\sqrt{2}$	
$q^{(1)}$	$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$		$\begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \end{pmatrix}$		$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$	
$\nu^{(1)}$	$(1,0,1,0) \begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \end{pmatrix}$		$(\frac{1}{2},0,\frac{1}{2},0) \begin{pmatrix} \frac{3}{2} \\ 0 \\ 1 \\ 0 \end{pmatrix}$		$\frac{1}{2}(1,0,1,0) \begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \end{pmatrix}$	
$= (q^{(1)})^T \underbrace{Aq^{(1)}}_{=z^{(2)}}$	$= 3+2=5$		$= \frac{3}{4}+\frac{1}{2}=\frac{5}{4}=1.25$		$= \frac{1}{2}(3+2)=\frac{5}{2}=2.5$	
$z^{(2)} = Aq^{(1)}$	$\begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \end{pmatrix}$		$\begin{pmatrix} \frac{3}{2} \\ 0 \\ 1 \\ 0 \end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{2}} \\ 0 \\ \sqrt{2} \\ 0 \end{pmatrix}$	

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$$A = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = \frac{3+\sqrt{5}}{2} \approx 2.618, \lambda_{2,3} = 1, \lambda_4 = \frac{3-\sqrt{5}}{2} \approx 0.382, \Rightarrow x_1 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5}-1 \\ 0 \end{pmatrix}, x_4 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5}+1 \\ 0 \end{pmatrix}$$

	$\|\cdot\|_\infty$	$\|\cdot\|_1$	$\|\cdot\|_2$
$\|z^{(2)}\|$	3	$\frac{3}{2}+1 = \frac{5}{2}$	$\sqrt{\frac{9}{2}+2} = \sqrt{\frac{13}{2}}$
$q^{(2)}$	$\begin{pmatrix} 1 \\ 0 \\ \frac{2}{3} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{3}{2}\cdot(\frac{5}{2})^{-1} \\ 0 \\ \frac{2}{5} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} \\ 0 \\ \frac{2}{5} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{3}{\sqrt{2}}\cdot(\frac{\sqrt{13}}{\sqrt{2}})^{-1} \\ 0 \\ \sqrt{2}\cdot(\frac{\sqrt{13}}{\sqrt{2}})^{-1} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{13}}\begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \end{pmatrix}$
$\nu^{(2)}$	$(1,0,\frac{2}{3},0)\begin{pmatrix} \frac{8}{3} \\ 0 \\ \frac{5}{3} \\ 0 \end{pmatrix}$	$(\frac{3}{5},0,\frac{2}{5},0)\begin{pmatrix} \frac{8}{5} \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\frac{1}{13}(3,0,2,0)^T\begin{pmatrix} 8 \\ 0 \\ 5 \\ 0 \end{pmatrix}$
$= (q^{(2)})^T \underbrace{Aq^{(2)}}_{=z^{(3)}}$	$= \frac{8}{3}+\frac{10}{9} = \frac{34}{9} \approx 3.778$	$= \frac{24}{25}+\frac{2}{5} = \frac{34}{25} = 1.36$	$= \frac{(24+10)}{13} = \frac{34}{13} \approx 2.615$
$z^{(3)} = Aq^{(2)}$	$\begin{pmatrix} \frac{8}{3} \\ 0 \\ \frac{5}{3} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{8}{5} \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\frac{1}{\sqrt{13}}\begin{pmatrix} 8 \\ 0 \\ 5 \\ 0 \end{pmatrix}$
$\|z^{(3)}\|$	$\frac{8}{3}$	$\frac{8}{5}+1 = \frac{13}{5}$	$\sqrt{\frac{64}{13}+\frac{25}{13}} = \sqrt{\frac{89}{13}}$
$q^{(3)}$	$\begin{pmatrix} 1 \\ 0 \\ \frac{5}{8} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0.625 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{8}{13} \\ 0 \\ \frac{5}{13} \\ 0 \end{pmatrix} \approx \begin{pmatrix} 0.615 \\ 0 \\ 0.385 \\ 0 \end{pmatrix}$	$\sqrt{\frac{13}{13\cdot89}}\begin{pmatrix} 8 \\ 0 \\ 5 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{89}}\begin{pmatrix} 8 \\ 0 \\ 5 \\ 0 \end{pmatrix} \approx \begin{pmatrix} 0.848 \\ 0 \\ 0.530 \\ 0 \end{pmatrix}$

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$$A = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \lambda_1 = \frac{3+\sqrt{5}}{2} \approx 2.618, \lambda_{2,3} = 1, \lambda_4 = \frac{3-\sqrt{5}}{2} \approx 0.382, \Rightarrow x_1 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5}-1 \\ 0 \end{pmatrix}, x_4 = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5}+1 \\ 0 \end{pmatrix}$$

	$\|\cdot\|_\infty$	$\|\cdot\|_1$	$\|\cdot\|_2$
$\nu^{(3)}$	$(1,0,\frac{5}{8},0)\begin{pmatrix} \frac{21}{8} \\ 0 \\ \frac{13}{8} \\ 0 \end{pmatrix}$	$(\frac{8}{13},0,\frac{5}{13},0)\begin{pmatrix} \frac{21}{13} \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\frac{1}{89}(8,0,5,0)\begin{pmatrix} 21 \\ 0 \\ 13 \\ 0 \end{pmatrix}$
$= (q^{(3)})^T \underbrace{Aq^{(3)}}_{=z^{(4)}}$	$= \frac{21}{8} + \frac{65}{64} = \frac{233}{64} \approx 3.641$	$= \frac{168}{169} + \frac{5}{13} = \frac{233}{169} \approx 1.379$	$= \frac{168+65}{89} = \frac{233}{89} \approx 2.618$
$z^{(4)} = Aq^{(3)}$	$\begin{pmatrix} \frac{21}{8} \\ 0 \\ \frac{13}{8} \\ \frac{3}{0} \end{pmatrix}$	$\begin{pmatrix} \frac{21}{13} \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\frac{1}{\sqrt{89}}\begin{pmatrix} 21 \\ 0 \\ 13 \\ 0 \end{pmatrix}$
$\|z^{(4)}\|$	$\frac{21}{8}$	$\frac{21}{13} + 1 = \frac{34}{13}$	$\sqrt{\frac{441}{89} + \frac{169}{89}} = \sqrt{\frac{610}{89}}$
$q^{(4)}$	$\begin{pmatrix} 1 \\ 0 \\ \frac{13}{21} \\ 0 \end{pmatrix} \approx \begin{pmatrix} 1 \\ 0 \\ 0.619 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{21}{34} \\ 0 \\ \frac{13}{34} \\ 0 \end{pmatrix} \approx \begin{pmatrix} 0.618 \\ 0 \\ 0.382 \\ 0 \end{pmatrix}$	$\sqrt{\frac{89}{89\cdot610}}\begin{pmatrix} 21 \\ 0 \\ 13 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{610}}\begin{pmatrix} 21 \\ 0 \\ 13 \\ 0 \end{pmatrix} \approx \begin{pmatrix} 0.850 \\ 0 \\ 0.526 \\ 0 \end{pmatrix}$
$\nu^{(4)}$	$(1,0,\frac{13}{21},0)\begin{pmatrix} \frac{55}{21} \\ 0 \\ \frac{34}{21} \\ 0 \end{pmatrix}$	$(\frac{21}{34},0,\frac{13}{34},0)\begin{pmatrix} \frac{55}{34} \\ 0 \\ 1 \\ 0 \end{pmatrix}$	$\frac{1}{610}(21,0,13,0)\begin{pmatrix} 55 \\ 0 \\ 34 \\ 0 \end{pmatrix}$
$= (q^{(4)})^T \underbrace{Aq^{(4)}}_{=z^{(5)}}$	$= \frac{55}{21} + \frac{442}{441} = \frac{1597}{441} \approx 3.621$	$= \frac{1155}{34^2} + \frac{13}{34} = \frac{1597}{1156} \approx 1.382$	$= \frac{1155+442}{610} = \frac{1597}{610} \approx 2.618$