Universität Duisburg-Essen
Computational Mechanics
Campus Essen

Fall Term 2010/11

Dr. S. Vanis

28. October 2010

Introduction to Numerical Methods
Tutorial 3

Exercise 1:
Interpolate the function $f(x) = e^x$ on the interval $[-1, 1]$ by a natural cubic spline function using the interpolating nodes $x_0 = -1, x_1 = 0$, and $x_2 = 1$.
Take your favorite plot program and plot your solution as well as the original function $f(x)$ into one diagram on the interval $[-1, 1]$.

Exercise 2:
Show that for the interpolating polynomial $p(x)$ of $f(x) = \frac{2}{(3+x)}$ using the interpolation nodes $x_0 = -2, x_1 = -1, x_2 = 1$, and $x_3 = 2$ the following estimate holds

$$\|f - p\|_\infty := \max_{x\in[-2,2]} |f(x) - p(x)| \leq 8$$

(*)-Exercise 3: (6 + 4 = 10 points)

(i) Show that the $(n + 1)$st derivative of $\omega(x)$ in Theorem 2.3 holds

$$\frac{d^{n+1}w(x)}{dx^{n+1}} = \frac{d^{n+1}}{dx^{n+1}}\Big(\prod_{i=0}^{n}(x - x_i)\Big) = (n + 1)!$$

(ii) Show that the expression

$$|||f||| := \Big(\int_a^b (f''(x))^2\, dx\Big)^{\frac{1}{2}} \;\; , \; f \in \mathcal{C}^2([a, b])$$

is a seminorm, i.e., show that it is a norm except for the property

$$|||f||| = 0 \Rightarrow f \equiv 0.$$

Delivery: Thursday, 4. November 2010