Universität Duisburg-Essen
Computational Mechanics
Campus Essen

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Fall Term 2010/11

4. November 2010

Introduction to Numerical Methods
Tutorial 4

Exercise 1:
Compute the weights $\tilde{\alpha}_i$ of the Newton-Cotes formula for $n = 2$ and $n = 3$ and write down the obtained formulas, e.g., for $n = 1$ we have $\hat{I}(f) = \frac{b-a}{2}(f(a) + f(b))$.

Exercise 2:
All results should be given with 7 digits.

(i) Compute the following integral with the Trapezoidal rule, with the Simpson's rule and with Simpson's 3/8-rule, i.e., with the Newton-Cotes formulas for $n = 1, 2$, and 3.

$$\int_{-1}^{1} e^x \, dx$$

(ii) Compute the error of the numerical integration.

(iii) Compare the values of the error estimates with the actual error you computed in (ii).
Use therefore the formulas from the table in the lecture notes, i.e., $\frac{h^3}{12}f''(\eta)$ for $n = 1$, $\frac{h^5}{90}f^{(4)}(\eta)$ for $n = 2$, and $\frac{3h^5}{80}f^{(4)}(\eta)$ for $n = 3$. Note, in an exam such formulas would not be given.

(*)-Exercise 3: (4 + 4 = 8 points)

(i) For the numerical approximation of

$$I(f) = \int_{0}^{1} f(x) \, dx$$

a quadrature rule of the form

$$\tilde{I}(f) = b_0 f(x_0) + b_1 f(x_1) \quad \text{with} \quad x_0 = \frac{1}{2} + \frac{\sqrt{3}}{6}, x_1 = \frac{1}{2} - \frac{\sqrt{3}}{6},$$

should be used.
Compute b_1 and b_2 such that the obtained quadrature rule is exact for polynomials of at least degree 2.

(ii) Let $h > 0$ be given and define

$$T := \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, x + y \leq h\}.$$

Furthermore let $a_0 := (0, 0), a_1 := (h, 0), a_2 := (0, h)$, and $s := (\frac{h}{3}, \frac{h}{3})$ be given.

a) Compute the weights ω_0, ω_1, and $\omega_2 \in \mathbb{R}$ such that the quadrature rule

$$\hat{I}(f) := \sum_{i=1}^{3} \omega_i f(a_i) \approx I(f) := \int_T f(x, y)\ dy\ dx$$

integrate polynomials of degree one in two variables exactly.

b) Show that there exists a constant $\omega \in \mathbb{R}$ such that

$$I(p) = wp(s)$$

for all polynomials of degree 1 in two dimensions.

Delivery: Thursday, 11. November 2010