Universität Duisburg-Essen
Computational Mechanics
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11. November 2010

Introduction to Numerical Methods
Tutorial 5

Exercise 1:
Determine the smallest number of intervals $n, m \in \mathbb{N}$ which are needed to approximate

$$I(f) := \int_0^1 e^x \, dx$$

up to an accuracy of $5 \cdot 10^{-4}$ when using

(i) the composite Trapezoidal rule,

(ii) the composite Simpson's rule.

Exercise 2:
Use the composite Trapezoidal rule with $n = 6$ to approximate the integral

$$I(f) := \int_2^{2.3} \sqrt{x} \, dx.$$

Compute the actual error and an estimate for the error between $I(f)$ and the approximation with the composite Trapezoidal rule.

Exercise 3:
This exercise will be shifted to Tutorial 6 if Theorem 3.5 was not part of the lecture on the 11. November.

In this exercise we will compare the Trapezoidal rule with the Gaussian quadrature.
Therefore you should determine the approximation to

$$I(f) := \int_0^1 \frac{1}{\sqrt{x}} \, dx$$

by

(i) using the composite Trapezoidal rule for

$$I_k := \int_{\frac{1}{k}}^{1} \frac{1}{\sqrt{x}}\, dx \quad k = 3, 4, 5, 6$$

with $n = k - 1$ intervals,

(ii) using Gauß-Legendre integration with $n = 4$. Therefore you need the values of the nodes x_i and the weights A_i for the Gauß-Legendre integration given in the table below for $n = 1, 2, 3$, and 4.

n	i	$x_i^{(n)}$	$A_i^{(n)}$
1	1	0	2
2	1	$\sqrt{\frac{1}{3}} \approx 0.5773503$	1
	2	$-\sqrt{\frac{1}{3}} \approx -0.5773503$	1
3	1	$\sqrt{\frac{3}{5}} \approx 0.7745967$	$\frac{5}{9} \approx 0.5555556$
	2	0	$\frac{8}{9} \approx 0.8888889$
	3	$-\sqrt{\frac{3}{5}} \approx -0.7745967$	$\frac{5}{9} \approx 0.5555556$
4	1	$\sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}} \approx 0.8611363$	$\frac{18-\sqrt{30}}{36} \approx 0.3478548$
	2	$\sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}} \approx 0.3399810$	$\frac{18+\sqrt{30}}{36} \approx 0.6521455$
	3	$-\sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}} \approx -0.3399810$	$\frac{18+\sqrt{30}}{36} \approx 0.6521455$
	4	$-\sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}} \approx -0.8611363$	$\frac{18-\sqrt{30}}{36} \approx 0.3478548$

Compare both results with the actual result if you can analytically determine it.

(*)-Exercise 4: (3 + 2 + 3 = 8 points)

(i) Show how the composite Simpson's rule, equation (3.5) in the lecture notes, can be derived/ deduced from Simpson's rule, equation (3.3).

Hint: Look at the derivation/ deduction of the composite Trapezoidal rule and proceed in an analog way.

(ii) Show 1. from Theorem 3.4 by induction, i.e., show that $p_i \in \mathcal{P}_i$ for all $i = 0, \ldots, n$ with p_i as defined in Theorem 3.4.

(iii) Show 2. from Theorem 3.4 by induction. Here, it is sufficient to show that $< p_i, p_k >= 0$ for all $k = 0, \ldots, (i-1)$, and $i \in \mathbb{N}$.

Delivery: 18. November 2010