Universität Duisburg-Essen
Computational Mechanics
Campus Essen

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Dr. S. Vanis

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Introduction to Numerical Methods
Tutorial 6

Exercise 1:
In this exercise we will compare the Trapezoidal rule with the Gaussian quadrature.
Therefore you should determine the approximation to the integral

$$I(f) := \int_0^1 \frac{1}{\sqrt{x}}\, dx$$

(i) using the composite Trapezoidal rule with $n = k - 1$ intervals for

$$I_k := \int_{\frac{1}{k}}^1 \frac{1}{\sqrt{x}}\, dx \quad k = 3, 4, 5, 6 \quad ,$$

(ii) using Gauß-Legendre integration with $n = 4$. Therefore you need the values of the nodes x_i and the weights A_i for the Gauß-Legendre integration given in the table below for $n = 1, 2, 3$, and 4.

n	i	$x_i^{(n)}$	$A_i^{(n)}$
1	1	0	2
2	1	$\sqrt{\frac{1}{3}} \approx 0.5773503$	1
	2	$-\sqrt{\frac{1}{3}} \approx -0.5773503$	1
3	1	$\sqrt{\frac{3}{5}} \approx 0.7745967$	$\frac{5}{9} \approx 0.5555556$
	2	0	$\frac{8}{9} \approx 0.8888889$
	3	$-\sqrt{\frac{3}{5}} \approx -0.7745967$	$\frac{5}{9} \approx 0.5555556$
4	1	$\sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}} \approx 0.8611363$	$\frac{18-\sqrt{30}}{36} \approx 0.3478548$
	2	$\sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}} \approx 0.3399810$	$\frac{18+\sqrt{30}}{36} \approx 0.6521455$
	3	$-\sqrt{\frac{3}{7} - \frac{2}{7}\sqrt{\frac{6}{5}}} \approx -0.3399810$	$\frac{18+\sqrt{30}}{36} \approx 0.6521455$
	4	$-\sqrt{\frac{3}{7} + \frac{2}{7}\sqrt{\frac{6}{5}}} \approx -0.8611363$	$\frac{18-\sqrt{30}}{36} \approx 0.3478548$

Compare both results with the actual result if you can analytically determine it.

Exercise 2:
Compute the inverse of the matrix

$$\begin{pmatrix} 2 & 1 & -2 \\ 3 & 2 & 2 \\ 5 & 4 & 3 \end{pmatrix}$$

by computing the columns of the inverse as the solutions of the linear equation systems

$$Ax = e_i \quad \text{with} \quad e_i = (0, \ldots, 0, \underbrace{1}_{\text{i-th entry}}, 0, \ldots 0)^T$$

for $i = 1, 2, 3$. Solve the linear equation systems using the Gaussian elimination with column pivot search.

Exercise 3:
Compute the number of flops (floating point operations) needed to solve a linear system using Gaussian elimination without column pivot search with a real valued $(n \times n)$-matrix A and a right hand side $b \in \mathbb{R}^n$. (A flop was defined in the lectur in paragraph 2.2.)
You will need the following two formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n-1)}{6}.$$

(*)-Exercise 4: (5 + 5 = 10 points)
The linear equation system

$$\begin{pmatrix} 0.001 & -2.3 \\ -1.35 & 0.03 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2.295 \\ -6.72 \end{pmatrix}$$

has the solution $x_1 = 5$ and $x_2 = 1$.
To see the influence of errors occuring when a system is solved without column pivot search you should now solve the system yourself. But you are only allowed to use 3 digits, i.e., each number can be displayed in exponential form by

$$\pm 0.y_1 y_2 y_3 \cdot 10^E \quad (y_1, y_2, y_2 \in \{0, 1, \ldots, 9\}, y_1 \neq 0, E \in \mathbb{Z}).$$

If you would need more digits for a number it is cut without rounding, e.g., 12.45 becomes $0.124 \cdot 10^2$.
Compute the solution of the system with this representation of numbers using Gaussian elimination

(i) without using column pivot search,

(ii) using column pivot search.

Do you recognize a difference in the solutions?

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