Universität Duisburg-Essen
Computational Mechanics
Campus Essen

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Introduction to Numerical Methods
Tutorial 7

Exercise 1:
Compute manually the solution of the linear equation system $Ax = b$ with

$$A = \begin{pmatrix} 25 & 10 & 25 & 15 \\ 10 & 13 & 16 & -3 \\ 25 & 16 & 33 & -15 \\ 15 & -3 & -15 & 12 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} -5 \\ 10 \\ 23 \\ 15 \end{pmatrix}$$

using LU-decomposition with column pivot search.
Make sure that your result is correct by multiplying L with U.

(*)-Exercise 2: (7 + 2 + 3 = 12 points)
Consider the elimination matrices L_i needed in the LU-decomposition.
Show that

(i) $L'_i := P^T_{i+1} L_i P_{i+1}$ is the matrix L_i with permuted rows as caused by P_{i+1},

(ii) that the inverse matrix of L_j is given by

$$L_j^{-1} = \begin{pmatrix} 1 & 0 & \dots & \dots & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & & & & \vdots \\ 0 & \dots & 1 & 0 & \dots & \dots & 0 \\ 0 & \dots & l_{j+1,j} & 1 & 0 & \dots & 0 \\ \vdots & & \vdots & & \ddots & & \vdots \\ 0 & \dots & l_{nj} & 0 & \dots & 0 & 1 \end{pmatrix},$$

(iii) and that for a all nonsingular matrices $A \in \mathbb{R}^{n \times n}$ the decomposition $A = LU$ is uniquely defined. From this it follows directly that $PA = LU$ is uniquely defined, too.

Programming-Exercise 1: (delivery date: 9. December 2010, 15 points)
Write a program which performes for a given matrix $A \in \mathbb{R}^{n \times n}$ a LU-decomposition with coulmn pivot search and solves the linear equation system $Ax = b$ when an additional right hand side $b \in \mathbb{R}^n$ is given.
Use your prorgram to solve the systems $Ax = b$ given by

(i) the example in exercise 1,

(ii) the matrix $A = (a_{ij})_{i,j=1..n}$ with $a_{ij} = \dfrac{1}{(i+j-1)}$

and the vector $b = (b_i)_{i=1,..,n}$ with $b_i = \sum_{j=1}^{n} \dfrac{1}{(i+j-1)}$,

(iii) the matrix $A = (a_{ij})_{i,j=1..n}$ with

$$a_{ij} = \begin{cases} \dfrac{1}{(i+j-1)} & i \neq j \\ 1 & i = j \end{cases}$$

and the vector $b = (b_i)_{i=1,..,n}$ with $b_i = 1 + \sum_{\substack{j=1 \\ i \neq j}}^{n} \dfrac{1}{(i+j-1)}$.

Show using Linear Algebra that the solutions you obtain are correct.
Test your program for different $n \in \mathbb{N}$, e.g., $n = 5, 10, 25, 100$.
Program the forward- and backward-substitution by yourself and do not use the matlab routine for solving linear system.

Hint: Start with an LU-decomposition without pivot-search and then implement the pivot search in a second step.
Don't implement the pivot search and the elimination as matrix-matrix-products.

You can store L and U in A if you like, cf., the lecture notes concerning the advantages of this proceeding, but you can also store them separately and then test your result by computing $LU - PA$.

Delivery: 2. December 2010