Universität Duisburg-Essen
Computational Mechanics
Campus Essen

Fall Term 2010/11

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2. December 2010

Introduction to Numerical Methods
Tutorial 8

Exercise 1:

(i) Proof the norm properties for induced norms, i.e., proof 1. from Corollary 4.2 in the lecture.

(ii) Show that the "column-sum-norm" $\|A\|_1 = \max_{j=1,\ldots,n} \sum_{i=1}^{n} |a_{ij}|$ is induced by the "1-vector-norm" $\|x\|_1 = \sum_{i=1}^{n} |x_i|$.

Hint: Show that $\max_{j=1,\ldots,n} \sum_{i=1}^{n} |a_{ij}| \leq \|A\|_1 \leq \max_{j=1,\ldots,n} \sum_{i=1}^{n} |a_{ij}|$ which then implies equality.

Exercise 2:
Let the linear equation system $Ax = b$ with

$$A := \begin{pmatrix} 1.00 & 0.99 \\ 0.99 & 0.98 \end{pmatrix} \quad \text{and} \quad b := \begin{pmatrix} 1.99 \\ 1.97 \end{pmatrix}$$

be given. Furthermore assume that there exists a disturbed right hand side

$$b + \Delta b = \begin{pmatrix} 1.989903 \\ 1.970106 \end{pmatrix}.$$

(i) Compute manually the solutions

(1) x of $Ax = b$ and

(2) $x + \Delta x$ of $A(x + \Delta x) = (b + \Delta b)$.

(ii) Compute the relative errors $\frac{\|\Delta x\|_\infty}{\|x\|_\infty}$, $\frac{\|\Delta b\|_\infty}{\|b\|_\infty}$ and the condition number of A, i.e., $\kappa_\infty = \|A\|_\infty \|A^{-1}\|_\infty$, with an accuracy of at most 8 digits.

(iii) Insert your result in the estimate

$$\frac{\|\Delta x\|_\infty}{\|x\|_\infty} \leq \kappa_\infty \frac{\|\Delta b\|_\infty}{\|b\|_\infty}.$$

(*)-Exercise 3: (4 + 2 + 4 + 4 = 14 points)
In this exercise you will develop the algorithm for the Cholesky decomposition. Furthermore you will show some technical details of this decomposition and use your results to compute the solution of a linear equation system.

(i) Assume that $A \in \mathbb{R}^{n\times n}$ with $n \in \mathbb{N}$ has a Cholesky decomposition, i.e., there exists a lower triangluar matrix with positive diagonal entries such that $A = LL^T$. Show how you can compute the entries of L under the assumption that L exists.

(ii) Show: If A has a Cholesky decomposition, than A is symmetric, i.e., $A = A^T$, and A is positive definit, i.e., for all $x \in \mathbb{R}^n \setminus \{0\}$ holds $x^T Ax > 0$.

(iii) Assume that $n \in \mathbb{N}$ and $n \geq 3$. Show for the first three diagonal entries of L, i.e., l_{11}, l_{22}, and l_{33}, that they exist since A is symmetric positive definit.

(iv) Compute using your Cholesky decomposition from point (i) manually the solution $x \in \mathbb{R}^n$ of the linear equation system

$$\begin{pmatrix} 4 & -2 & 0 & 2 \\ -2 & 5 & 6 & 3 \\ 0 & 6 & 10 & 3 \\ 2 & 3 & 3 & 30 \end{pmatrix} x = \begin{pmatrix} 6 \\ -9 \\ -2 \\ -56 \end{pmatrix}$$

Delivery: 9. December 2010