Universität Duisburg-Essen
Computational Mechanics
Campus Essen

Dr. S. Vanis

Fall Term 2010/11

9. December 2010

Introduction to Numerical Methods
Tutorial 9

Exercise 1:
In this exercise you have to develop linear equation systems and solve them with the least-squares method.

(i) Consider the following measurements

x	1	2	3	4
y	0.5	1.5	5	7

Let us assume that these measurements describe a process which can be displayed using a exponential function $f(x) = a \cdot e^{bx}$, where a and b are unknown coefficients. Find a approximation for a and b using the least-squares method.
Avoid using rounded numbers apart from the result.
Hint: Consider $\ln(f(x))$ to obtain a linear equation.

(ii) The water level due to the tides in the North Sea with respect to the time t (in hours) is given by

$$H(t) = h + a\sin(\frac{2\pi x}{12}) + b\cos(\frac{2\pi x}{12}),$$

where h, a, and b are unknown real constants. Let the following data be given

t	0	2	4	6	8	10
$H(t)$	1.0	1.6	1.4	0.6	0.2	0.8

Compute a good solution to h, a, and b using the least-squares method.

Exercise 2: Use the Gram-Schmidt orthonomalization algorithm to compute manually a orthonormal basis for the space spanned by

$$a_1 := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, a_2 := \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, a_3 := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \text{ and } a_4 := \begin{pmatrix} 2 \\ 1 \\ 0 \\ 2 \\ 3 \end{pmatrix}.$$

(*)-Exercise 3: ((2 + 2 + [3 + 2]) + (1 + 1 + 1) = 12 points)

(i) Let $A \in \mathbb{R}^{n\times n}$, i.e., A is a real $(n \times n)$-matrix. Show that the following things hold

(1) If A is positive definite, i.e., $x^T Ax > 0$ for all $x \in \mathbb{R}^n \setminus \{0\}$, then $a_{ii} > 0$ for all $i = 1, \dots, n$, i.e., all diagonal entries of A are positive.

(2) If A is skew symmetric, i.e., $A = -A^T$, then $a_{ii} = 0$ for all $i = 1, \dots, n$, i.e., all diagonal entries are zero.

(3) The following two propositions are equivalent

(a) A is skew symmetric

(b) $x^T Ax = 0 \quad \forall x \in \mathbb{R}^n \setminus \{0\}$

Hint: Show first that (a) implies (b), i.e., if A is skew symmetric then $x^T Ax = 0 \ \forall x \in \mathbb{R}^n \setminus \{0\}$, and then that (b) implies (a).

(ii) Draw manually pictures of the following subsets of the twodimensional real vector space

(1) $\{x \in \mathbb{R}^2 : \|x\|_\infty = 1\}$,

(2) $\{x \in \mathbb{R}^2 : \|x\|_1 \leq 1\}$, and

(3) $\{x \in \mathbb{R}^2 : 1 < \|x\|_2 < 2\}$.

Draw one picture for each set, i.e., **don't** draw all sets into one picture.

Delivery: 16. December 2010